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Nonlinear electromagnetic spatial solitons in the resonant value of electromagnetic waves

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Abstract: *For such a multi-valued nonlinear magnetic response, the domains with different values of the magnetic permeability "excited" by the spatial soliton can be viewed as effective induced left-handed waveguides which make possible the existence of single- and multi-hump soliton structures. Due to the existence of such domains, the solitons can be not only symmetric, but also antisymmetric and even asymmetric. Formally, the size of an effective domain can be much smaller than the wavelength and, therefore, there exists an applicability limit for the obtained results to describe nonlinear waves in realistic composite structures.*

Keywords: soliton, symmetric, resonant, dielectric, magnetic

Introduction

First, we follow the original paper [1] and consider a two-dimensional composite structure consisting of a square lattice of the periodic arrays of conducting wires and split-ring resonators (SRR). We assume that the unit-cell size d of the structure is much smaller then the wavelength of the propagating electromagnetic field and, for simplicity, we choose the single-ring geometry of a lattice of cylindrical SRRs. The results obtained for this case are qualitatively similar to those obtained in the more involved cases of double SRRs. This type of microstructured materials has recently been suggested and built in order to create left-handed metamaterials with negative refraction in the microwave region [2].

The negative real part of the effective dielectric permittivity of such a composite structure appears due to the metallic wires whereas a negative sign of the magnetic permeability becomes possible due to the SRR lattice. As a result, these materials demonstrate the properties of negative refraction in the finite frequency band, $\omega_0 < \omega < min(\omega_p, \omega_{||m})$, where ω_0 is the eigenfrequency of the SRRs, $\omega_{||m}$ is the frequency of the longitudinal magnetic plasmon, ω_p is an effective plasma frequency, and ω is the angular frequency of the propagating electromagnetic wave, $(\mathcal{E}, \mathcal{H}) \sim (\mathbf{E}, \mathbf{H})\ exp\ (i\omega t)$. The split-ring resonator can be described as an effective LC oscillator (see, e.g. Ref. [7]) with the capacitance of the SRR gap, as well as an effective inductance and resistance. Similar to other nonlinear media, nonlinear lefthanded composite materials can support self-trapped electromagnetic waves in the form of *spatial solitons*. Such solitons possess interesting properties because they exist in materials with a hysteresis-type (multi-stable) nonlinear magnetic response. Below, we describe novel and unique types of single and multi-hump (symmetric, antisymmetric, or even asymmetric) backward-wave spatial electromagnetic solitons supported by the nonlinear magnetic permeability.

Nonlinear response of such a composite structure can be characterized by two different contributions. The first one is an intensity-dependent part of the effective dielectric permittivity of the infilling dielectric. For simplicity, we may assume that the metallic structure is embedded into a nonlinear dielectric with a permittivity that depends on the intensity of the electric field in a general form, $\epsilon_D = \epsilon_D\ (|E|^2)$. For results of calculations presented below, we take the linear dependence that corresponds to the Kerr-type nonlinear response [9].

Electromagnetic spatial solitons in waves resonant response

The second contribution into the nonlinear properties of the composite material comes from the lattice of resonators, since the SRR capacitance (and, therefore, the SRR eigenfrequency) depends on the strength of the local electric field in a narrow slot. The intensity of the local electric field in the SRR gap depends on the electromotive force in the resonator loop, which is induced by the magnetic field. Therefore, the effective magnetic permeability μ_{eff} depends on the macroscopic (average) magnetic field **H**, and this dependence can be found in the form [8]

$$\mu_{eff}(\mathbf{H}) = 1 + \frac{\mathbf{F}\omega^2}{\omega_{0NL}^2(\mathbf{H}) - \omega^2 + i\Gamma\omega} \qquad (1)$$

where

$$\omega_{0NL}^2(\mathbf{H}) = \left(\frac{c}{a}\right)^2 \frac{d_g}{\left[\pi h \epsilon_D(|\mathbf{E}_g(\mathbf{H})|^2)\right]} \qquad (2)$$

is the eigenfrequency of oscillations in the presence of the external field of a finite amplitude, h is the width of the ring, $\Gamma = c^2/2\pi\sigma a h$, for $h < \delta$, and $\Gamma = c^2/2\pi\sigma a\delta$, for $h > \delta$. It is important to note that Eq. (1) has a simple physical interpretation: The resonant frequency of the artificial magnetic structure depends on the amplitude of the external magnetic field and, in turn, this leads to the intensity-dependent function μ_{eff}.

Due to the high values of the electric field in the slot of SRR as well as resonant interaction of the electromagnetic field with the SRR lattice, the characteristic magnetic nonlinearity in such structures is much stronger then the corresponding electric nonlinearity. Therefore, *magnetic nonlinearity should dominate* in the composite metamaterials. More importantly, the nonlinear medium can be created by inserting nonlinear elements into the slots of SRRs, allowing an easy tuning by an external field.

The critical fields for switching between LH and RH states, shown in the Figs. 1 can be reduced to a desirable value by choosing the frequency close to the resonant frequency of SRRs. Even for a relatively large difference between the SRR eigenfrequency and the external frequency, as we have in Fig. 1(b) where $\Omega = 0.8$ (i.e. $\omega = 0.8\omega_0$), the switching amplitude of the magnetic field is $\sim 0.03E_c$. The characteristic values of the focusing nonlinearity can be estimated for some materials such as n-InSb for which $E_c = 200 V/cm$ [3]. As a result, the strength of the critical magnetic field is found as $H_{c1} \approx 1.6 A/m$. Strong defocusing properties for microwave frequencies are found in $Ba_xSr_{1-x}TiO_3$ (see Ref. [14] and references therein). The critical nonlinear field of a thin film of this material is $E_c = 4 \cdot 10^4 V/cm$, and the corresponding field of the transition from the LH to RH state [see Fig. 1 (c)] can be found as $H_c \approx 55.4 A/m$.

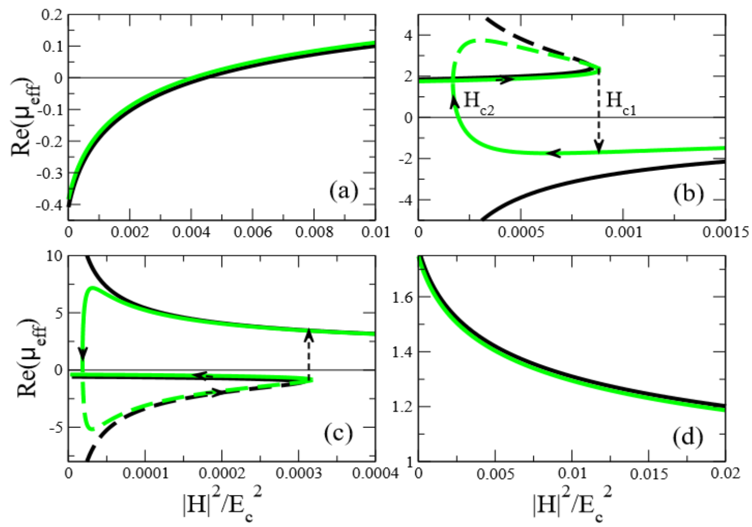

Figure 1: Real part of the effective magnetic permeability vs. intensity of the magnetic field: (a) $\Omega > 1$, $\alpha = 1$; (b) $\Omega < 1$, $\alpha = 1$, (c) $\Omega > 1$, $\alpha = -1$; and (d) $\Omega < 1$, $\alpha = -1$. Black – the lossless case ($\gamma = 0$), grey–the lossy case ($\gamma = 0.05$). Dashed curves show unstable branches.

The unique possibility of strongly enhanced effective nonlinearities in the left-handed metamaterials revealed here may lead to an essential revision of the concepts based on the linear theory, since the electromagnetic waves propagating in such materials always have a finite amplitude. At the same time, the engineering of nonlinear composite materials may open a number of their novel applications such as frequency multipliers, beam spatial spectrum transformers, switchers, limiters, etc.

Spatially localized TM-polarized waves that are described by one component of the magnetic field and two components of the electric field. Monochromatic stationary waves with the magnetic field component $H = H_y$ propagating along the z-axis and homogeneous in the y-direction, [$\sim \exp(i\omega t - ikz)$], are described by the dimensionless nonlinear Helmholtz equation where $\gamma = kc/\omega$ is a wavenumber, $x = x'\omega/c$ is the dimensionless coordinate, and x' is the dimensional coordinate. Different types of localized solutions of Eq. (1) can be analyzed on the phase plane $(H, dH/dx)$ (see, e.g., Refs.

[7]). First, we find the equilibrium points: the point (0,0) existing for all parameters, and the point $(0,H_1)$, where H_1 is found as a solution of the equation.

Below the threshold, i.e. for $\gamma < \gamma_{tr}$, where $\gamma_{tr}=\epsilon[1 + F\Omega^2/(1 - \Omega^2)]$, the only

Figures 1 and 2 summarize different types of nonlinear magnetic properties of the composite, which are defined by the dimensionless frequency of the external field $\Omega = \omega/\omega_0$, for both a *focusing* [Figs. 1, 2(a,b)] and a *defocusing* [Figs. 1, 2(c,d)] nonlinearity of the dielectric.

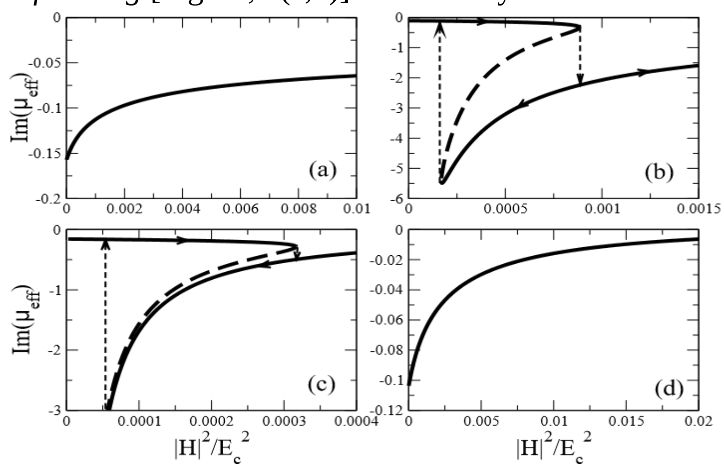

Figure 2: Imaginary part of the effective magnetic permeability vs. intensity of the magnetic field for $\gamma = 0.05$: (a) $\Omega > 1$, $\alpha = 1$; (b) $\Omega < 1$, $\alpha = 1$, (c) $\Omega > 1$, $\alpha = -1$; and (d) $\Omega < 1$, $\alpha = -1$. Dashed curves show unstable branches.

The unique possibility of strongly enhanced effective nonlinearities in the left-handed metamaterials revealed here may lead to an essential revision of the concepts based on the linear theory, since the electromagnetic waves propagating in such materials always have a finite amplitude. At the same time, the engineering of nonlinear composite materials may open a number of their novel applications such as frequency multipliers, beam spatial spectrum transformers, switchers, limiters, etc. equilibrium state (0,0) is a saddle point and, therefore, no finite-amplitude or localized waves can exist. Above the threshold value, i.e. for $\gamma > \gamma_{tr}$, the phase plane has three equilibrium points, and a separatrix curve corresponds to a soliton solution.

In the vicinity of the equilibrium state (0,0), linear solutions of Eq. (1) describe either exponentially growing or exponentially decaying modes. The equilibrium state $(0,H_1)$ describes a finite-amplitude wave mode of the transverse electromagnetic field. In the region of multi-stability, the type of the phase trajectories is defined by the corresponding branch of the multi-valued magnetic permeability. Correspondingly, different types of the spatial solitons appear when the phase trajectories correspond to the different branches of the nonlinear magnetic permeability.

The fundamental soliton is described by the separatrix trajectory on the plane $(H,dH/dx)$ that starts at the point (0,0), goes around the center point $(0,H_1)$, and then returns back; the corresponding soliton profile is shown in Fig. 1(a). More complex solitons are formed when the magnetic permeability becomes multi-valued and is described by several branches. Then, soliton solutions are obtained by switching between the separatrix trajectories corresponding to different (upper and lower) branches of magnetic permeability. Continuity of the tangential components of the electric and magnetic fields at the boundaries of the domains with different values of magnetic permeability implies that both H and dH/dx should be continuous. As a result, the transitions between different phase trajectories should be continuous.

Figures 1(b,c) show several examples of the more complex solitons corresponding to a single jump to the lower branch of $\mu(H)$ (dotted) and to the upper branch of $\mu(H)$ (dashed), respectively. The insets show the magnified domains of a steep change of the magnetic field. Both the magnetic field and its derivative, proportional to the tangential component of the electric field, are continuous. The shaded areas show the effective domains where the value of magnetic permeability changes. Figure 1(d) shows an example of more complicated multi-hump soliton which includes two domains of the effective magnetic permeability, one described by the lower branch, and the other one – by the upper branch. In a similar way, we can find more complicated solitons with different number of domains of the effective magnetic permeability.

We note that some of the phase trajectories have discontinuity of the derivative at $H = 0$ caused by infinite values of the magnetic permeability at the corresponding branch of $\mu_{\text{eff}}(H)$. Such a non-physical effect is an artifact of the lossless model of a left-handed nonlinear composite considered here for the analysis of the soliton solutions. In more realistic models that include losses, the region of multi-stability does not extend to the point $H = 0$, and in this limit the magnetic permeability remains a single-valued function of the magnetic field [1].

Conclusion

For such a multi-valued nonlinear magnetic response, the domains with different values of the magnetic permeability ”excited” by the spatial soliton can be viewed as effective induced left-handed waveguides which make possible the existence of single- and multi-hump soliton structures. Due to the existence of such domains, the solitons can be not only symmetric, but also antisymmetric and even asymmetric. Formally, the size of an effective domain can be much smaller than the wavelength and, therefore, there exists an applicability limit for the obtained results to describe nonlinear waves in realistic composite structures.

When the infilling dielectric of the structure displays *self-focusing nonlinear response*, we have $\Omega < 1$, and in such system we can find *dark solitons*, i.e. localized dips on the finite-amplitude background wave [8]. Similar to bright solitons, there exist both fundamental dark solitons and dark solitons with domains of different values of magnetic permeability. For self-defocusing nonlinearity and $\Omega < 1$, magnetic permeability is a single-valued function, and such a nonlinear response can support dark solitons as well, whereas for self-focusing dielectric, we have $\Omega > 1$ and no dark solitons can exist.

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