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EVALUATION OF WIND-INDUCED FATIGUE DAMAGE FOR EXISTING STRUCTURES CONSIDERING DIRECTIONAL EFFECTS

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Abstract. In many buildings, especially high rise ones, wind have been major problem by causing vibration as well as fatigue issues resulted from fluctuations of wind in random flowing. When the structures are designed appropriately enough, they are less likely to suffer from damage caused by wind induced fatigue. However, in some cases, the structures are under designed and fatigue damage can cause non negligible damage and components may start fatigue crack initiation when larger stress is concentrated in some parts. And this result in complete or partially failure of components of parts.

Keywords: Wind, induced fatigue, structure, Palmgren Miner,

1. INTRODUCTION

Calculation of fatigue damage is done through a model provided by Palmgren-Miner rule. For some definite cycle stress amplitude, the experienced load cycles n_{exp} is compared to the number of load cycles N_{fail} which leads to complete failure of steel components. These two variables are considered as random. The damage inflicted by some cycle of stress level is calculated as a ratio of number of experienced load cycles at this stress level and quantity of stress cycles leading it to complete failure.

2. RESEARCH METHODS

2.1. Probabilistic model for fatigue damage

Most of the mechanical components and components in structures are subjected to fatigue due to random loading as well as constant amplitude loading during their usage. In failure of such components fatigue is one of the primary reasons. Accumulation of damage is a complex and irreversible phenomenon, wherein the damage of the component under consideration gradually accumulates and over a period of time results in its complete rupture or failure. Therefore, this accumulation can be regarded as a measure of degradation in fatigue resistance of materials. Moreover, the damage accumulation is probabilistic in nature and degradation measure is believed to increase probabilistically with time.

In our case fatigue damage caused by wind actions is often related to vibrations, e.g. resonant excitations by vortex shedding. If the structure under consideration is designed well enough considering these effects of wind, the probability of its suffer from fatigue is low. However, when it is under-designed, damage imposed by fatigue is considerable.

Thus, the problem needs to consider probabilistic models to deal with the fatigue damage inflicting buildings over time. The two most widely used models for fatigue loading are S-N curve and Palmgren-Miner's damage accumulation models. The chapter elaborates these two models and gives basic equations.

2.2. Random S-N curves

A performance of a specific material under cyclic loading can be described by a S-N curve, which plots the cyclic stress S versus number of cycles N until its complete failure. It is also known as Wöhler curve. In Figure 1 such curve is depicted in which steel's fatigue life cycles in logarithmic scale is plotted against stresses corresponding to these cycle numbers.

Data representing fatigue are often given in the form of a median S-N curve, a log-log plot of cyclic stress or strain s versus the median fatigue life N expressing fatigue life of material till failure. The concept can be extended with the p quantile S-N curves, also called S-N-P curves, a generalization that relates the p-quantile of fatigue life to the applied stress or strain. Thus, each curve represents a constant probability of failure p, as a function of s. Normally it used the .05 and .95 quantile S-N curves to illustrate the variability of fatigue life, unless otherwise specified. In problem, random variables are specified for non-exceedance probabilities of 0.1%, 10%, 50%, 90% and 99.9% [3]. With this basis slight extrapolations are possible, i.e. the probabilistic model ranges from p=.001 to p=0.999 which corresponds to a reduced normally distributed variable from z from -3.090, to +3.090, [4] depicted in Figure 15. The integration must be calculated from -3.090 till +3.09 at every reference wind speed levels rising from 0.01 m/s till 30 m/s.

The increasing range for starts from 0.25, which corresponds to the ultimate limit failure for an increasing load, and includes full range of low- and high-cycle fatigue.

Most *S*–*N* curves are determined in laboratories where test specimens are subjected to constant amplitude until failure. Sinusoidal stresses are applied on a specimen by a testing machine and the cycles are counted until its failure. The process is also known as coupon testing. The *S*–*N* curves are derived by their mean fatigue life and the standard deviation of log *N*.

The general expression of the S-N curves for steel bolts is given as:

$$\log(S) = a + b \cdot \arctan\left(\frac{1}{\nu} \cdot (log\,\Omega - log(N))\right) \tag{1-a}$$

The parameters Ω and ν are identified as deterministic values within the goniometric model. The random coefficients a and b are available for the non-exceedance probabilities 0.1%, 10%, 50%, 90%, 99.9%. In order to develop approximations for and b for any value of p in the range from 0.001 to 0.999, these non-exceedance probabilities should be substituted with corresponding reduced variate of the standardized normal distribution.

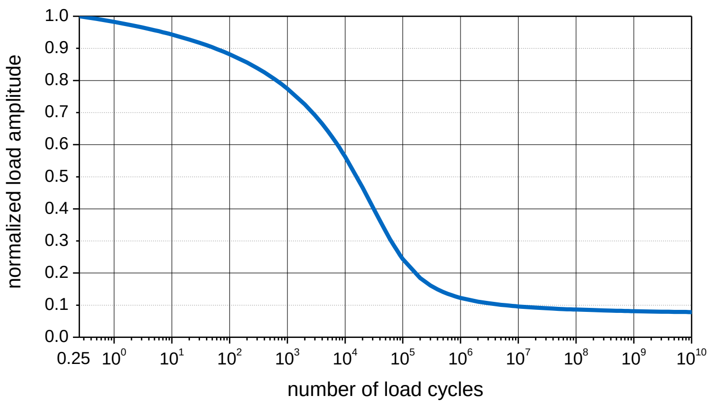


Figure 1. S-N (Wohler curve)

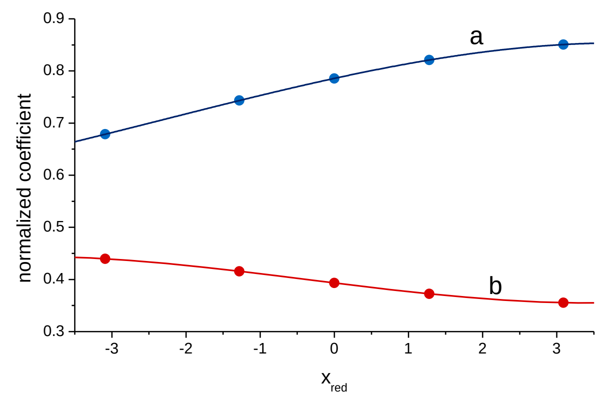


Figure 2. Approximations of a and b. [3]

$$a = 0.78573 + 0.03089g - 0.00221g^2 - 3.18796 - 4g^3 \tag{1-b}$$

$$b = 0.39377 - 0.01755g + 4.36754g^2 + 4.09507 - 4g^3$$

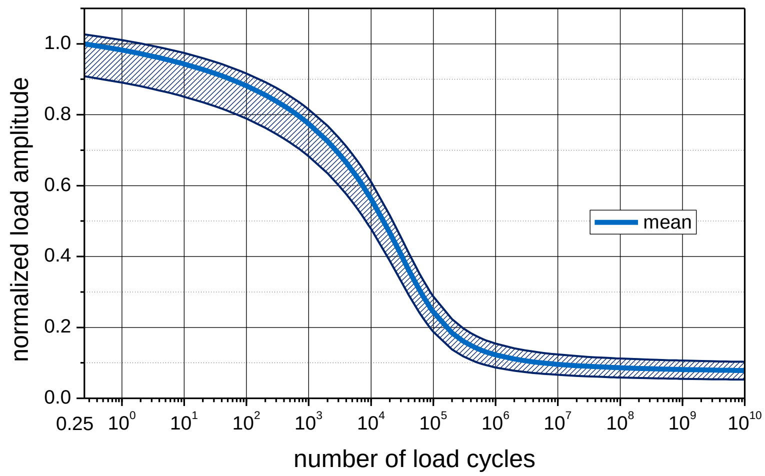


Figure 3. S-N relation from 0.001 to 0.999, i.e. the 99.8%-confidence [3]

In order to find N (total number of cycles under S stress till failure) we need to solve the equation with respect to S.

$$\arctan(\frac{1}{\nu} \cdot (log\Omega - \log N)) = \frac{\log(S) - a}{b} \quad (2)$$

Applying tan operator to both sides and solving for log(N) results in following equation:

$$\log(N) = -\tan\left(\frac{log(S) - a}{b}\right) \cdot \nu + \log \Omega \quad (3)$$

Finally, to obtain N the resulting left part of equation will be raised to 10 th power, since it is used in damage accumulation calculation which will be presented in next chapter.

2.3. Damage accumulation – Palmgren Miner

Palmgren Miner rule is one of the most commonly used damage accumulation equation used for failure resulting from fatigue.

$$\sum_{i=1}^{k} \frac{n_i}{N_i} = C \quad (4)$$

As one can notice from equation, Miner's Rule states that there are k different stress levels and the number of total stresses till failure of material occurs at the i-th stress S_i, is N_i. And the number of experienced cycles at that stress level S_i is n_i. The ratio in summation refers simply just fraction of damage that material undergoes, i.e. the amount of material life that is used up the stress level S_i. When the sum of damage fractions starts to be greater than 1.0, the failure of material occurs.

The experienced load cycles are analyzed based on wind tunnel experiments applying HCM-rain flow counting method. The basic result is a matrix of the number of load cycles for the two parameters mean value and amplitude. It is required to reduce the matrix to a vector of cycle numbers depending on only the amplitude in order to apply a simplified damage model Miner rule.

But these experienced load cycles have been counted assuming a design wind speed v_{ref}=25.9 m/s. They need to be transformed into another reference wind speed levels, since in the task it is required to alternate the reference wind speed from 0.01 m/s till 30 m/s. The transformation to other wind speed levels is done as follows:

$$n_k(\overline{R}, \Delta R) = n(\overline{c}_R, \Delta c_R) \cdot \frac{v_{ref}}{v_{des}} \tag{5}$$

v_{des} – is a design wind speed which has been used for counting the cycle histogram

v_{ref} – wind speed level at k-th hour of the working life.

The corresponding information on the amplitudes of R are specified as:

$$\overline{R} = \frac{1}{2} \cdot \overline{c}_R \cdot \rho_k \cdot v_k^2, \qquad \Delta R = \frac{1}{2} \cdot \Delta c_R \cdot \rho_k \cdot v_k^2 \tag{6}$$

ρ_k – air density in the k-th hour of the working life. For simplification it is take as ρ=1.25 kN/m^2.

2.4. Simulation strategy for confined ensembles

The problem, requires to create not just one or two correlated variables but a matrix of such variables. In calculating damage with taking directionality effects of wind climate into account, scale and shape variables of Weibull distribution for 36 directions should be generated. Using 3 correlation matrices of these variables and combining them it is possible to generate a 72x72 correlation matrix, from which it is possible to generate 72 random variables, half of which are scale parameters and other half are shape parameters.

Such random generation of variables can be achieved using Cholesky decomposition method in an efficient way.

2.5 Cholesky decomposition.

Cholesky decomposition is a powerful numerical technique that is mainly used in linear algebra in order to simplify algebraic operation with matrices which significantly reduces computation costs. Cholesky decomposition or factorization is a decomposition of positive definite matrix into a product of lower triangular matrix and its conjugate transpose matrix.

$$A = LL^T \tag{7}$$

L is lower triangular matrix with positive diagonal elements and is called Cholesky factor of A. The elements of lower triangular matrix can be found using these calculations:

$$l_{kk} = \sqrt{a_{kk} - \sum_{j=1}^{k-1} l_{kj}^2} \tag{8}$$

l_{kk} is a diagonal elements of lower triangular matrix.

The elements below diagonal elements are found in this way:

$$l_{ik} = \frac{1}{l_{kk}}\left(a_{ik} - \sum_{j=1}^{k-1} l_{ij} l_{kj}\right) \tag{9}$$

After finding all elements of lower triangular matrix one can easily generate an array of variables of n size for m year. To accomplish this task, firstly the matrix of random variables R following normal distribution with 0 mean and 1 standard deviation should be generated.

$$R_{ij} \sim N(0,1),\ i = 1, m;\ j = 1, n \tag{10}$$

Then this matrix is multiplied to the lower triangular matrix of A to obtain C matrix whose row variables of are correlated with correlation coefficients stated in matrix A.

$$C = RL \tag{11}$$

Finally, this C matrix has elements correlated along the row and using this method it is possible to generate n number of year Weibull parameters as required in damage scenario.

However, this method cannot provide full solution when the number of years of observation for variables is less than size of correlation matrix. To obtain the full matrix of decomposition some algebraic linear system of equations should be solved.

3. RESEARCH RESULTS

3.1 Linear system of equations.

The correlation matrix A has dimension 72-72 after combine all three correlation matrices of Weibull parameters. However, the observation years are from 1952 till 2017 making overall 66 years. This is less than the size of correlation matrix which results in incompetency of Cholesky decomposition method and requires further approaches to obtain correlated variables. [5]

One of the ways could be, firstly decomposing the correlation matrix up to its reduced size of *nred=nyear-1*. In the case of 66 years *nred* becomes 65. So the A matrix is reduced into *Ared* from 72-72 size to 65-65 dimension for Cholesky decomposition to be applicable. The linear weighting coefficients for the 7 dependent variables are obtained from a linear system of equations.

$$Ared * cvec = bvec \tag{12}$$

cvec – is linear weighting coefficients to be solved with size 65,
Ared – reduced matrix of correlation with 65-65 dimension,
bvec – correlations of independent variables with respective depending variables.
Strategy to generate 72 correlated variables is firstly making reduced correlation matrix *Ared elements* correlated. To do this Cholesky decomposition is applied to this matrix and a matrix of *nyear*-65 size

whose elements follow standard normal distribution is generated for some. Here, *nyear* is any number that represents the years for which the correlated variables are being generated.

Then, the generated matrix is multiplied by the reduced correlation matrix after application of Cholesky decomposition scheme. This way, we could obtain first 65 correlated variables.

In order to obtain the last 7 correlated elements, it is required to solve the above linear system of equations. To solve this linear equation, we decompose the matrix into upper and lower parts and apply forward and backward substitution. After finding *cvec* coefficients we can finally produce all correlated data.

However, these variables are from standard normal distribution with mean 0 and standard deviation 1. They need to be transformed by multiplying the standard deviation and adding mean.

3.2.Object of the study – industrial building with box shape

To learn the impacts of wind on buildings many wind experiments have been made by engineers and in our case also we use the data and experiment results of wind tunnel experiments done on box-shaped building. The building is doubly symmetric and therefore only the wind direction sectors span the space from 0^{o} till 90^{o}.

Overall the building is divided into 4 hypothetical cross sections over which the pressure induced by wind is measured. These sections are at the following relative lengths of the building: 0.025, 0.125, 0.25, 0.5. And along these cross sections 23 taps are points where these air pressures are observed and monitored. These taps are connected to sensors by tubes of 300 mm length. The flow direction of the wind is changed with 10^{o} step size. A set of 10 independent runs are obtained for each flow direction as well as cross section. Every run durates 120 minutes in full scale under design wind condition. A floor mounted tap is used to remove the background noise from e.g. the motor; the noise from the fan is removed by digital filter techniques.

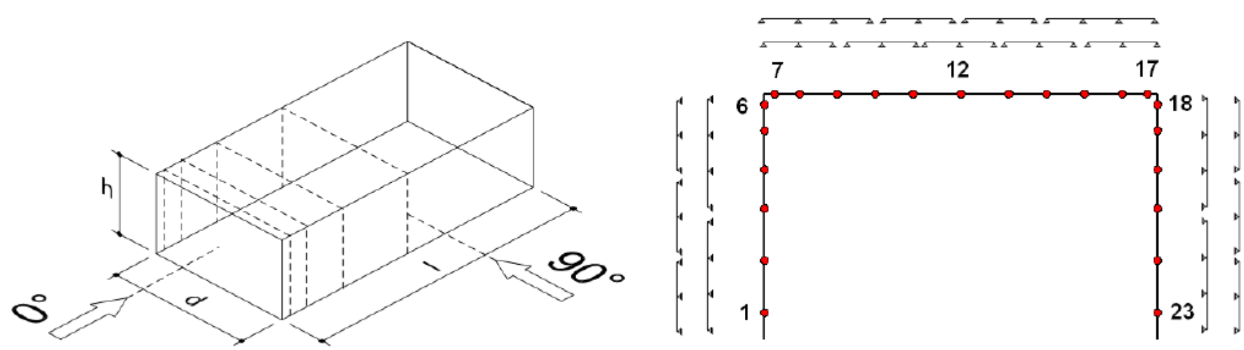


Figure 4. Wind tunnel model taps (h/d/l = 0.6 / 1.0 / 2.0) with position of the measured cross sections and position of the taps, possible arrangements of two- and three-field systems to cover the building envelope [6]

The focus of this study lies on the support reactions of trapezoidal sheeting elements. An example of such most common structural systems are continuous beams with two or three fields. The structural responses for the support reactions are obtained by convolving the instantaneous pressure distribution with the respective influence lines. Figure 24 shows the possible arrangements of two- and three-field systems to cover the building envelope.

3.3. Wind tunnel data – load cycle vectors for different taps and wind direction

In order to calculate the damage accumulation, we need to be aware of experienced wind load cycles, since as stated in above chapters Palmgren-Miner rule is used.

These experienced load cycles are analyzed based on wind tunnel experiments and through applying HCM-rainflow counting method. The basic result is a matrix of load cycles for the two parameters mean value and amplitude. Matrix consist of 100 rows and 10 columns. 10 columns refer to different flow directions. For each tap in cross section separate load cycle matrix data is present, making overall 23 files. To retrieve data for the load cycles for other flow directions one can use the double symmetry of box shaped building and easily find the remaining data. For example, if tap 5 is under analyze and the data from experiment provides only the angles of wind attack from 0^o to 90^o. Starting from 100^o the data will be symmetric along transverse cross section the building:

$$ncycmat_{tap5}(i, idir + 10) = ncycmat_{tap5}(i, 10 - idir), \ \ idir = 1{,}8 \qquad (13)$$

In this way, the half flow direction space data is ready. For other half again we use the symmetry. If looked from 180^o direction Tap 19 is located in the same spot of tap 5 when it is spotted from 0^o direction. Thus the load cycles from 180^o till 270^o is found using way and the last quartile directions of wind is found in the same way as done for directions of wind from 90^o till 170^o.

$$ncycmat_{tap19}(i, idir + 28) = ncycmat_{tap19}(i, 28 - idir), \ idir = 1{,}8 \qquad (14)$$

A more generalized result is obtained by sorting the cycles into classes, leading to the number of cycles per class which is defined by a range of mean values and double amplitudes, respectively. For the comparison of different support reactions, it is reasonable to normalize the classes with the respective standard deviation. Finally, the results from 10 runs can be averaged and normalized to a single hour under the reference wind speed condition. The number of load cycles in the k-th hour of the working life is obtained for a specific support reaction R as follows:

$$n_k(\overline{R}, \Delta R) = n(\overline{c}_R, \Delta c_R) \cdot \frac{v_{ref}}{v_{des}} \qquad (15)$$

vref - reference wind speed which has been used for counting the cycle histogram

vk - wind speed in the k-th hour of the working life

The corresponding information on the amplitudes of R are specified as:

$$\overline{R} = \frac{1}{2} \cdot \overline{c}_R \cdot \rho_k \cdot v_k^2, \qquad \Delta R = \frac{1}{2} \cdot \Delta c_R \cdot \rho_k \cdot v_k^2 \qquad (16)$$

ρ_k - air density in the k-th hour of the working life

Furthermore, other parameters cmaxmean and cmaxsdev to define design aerodynamic coefficient have been observed in 23 taps and written in files. This coefficient corresponds to 78 per cent fractile thus final design aerodynamic coefficient takes the following form:

$$cdes_{tap5}(id) = cmaxmean_{tap5}(id) + 0.63 \cdot cmaxsdev_{tap5}(id),\ id = 1{,}36 \tag{17}$$

Here also only information from id=1, 9 is present and the procedure to identify the remaining data is the same as done for wind load cycles, i.e. using the symmetry.

Aerodynamic coefficient is changed according to different under design states: 20%, 30%, 40% under designs. In case of 20% under design the coefficient becomes:

$$cdes = 0.8 \cdot cdes \tag{18}$$

3.4 Results of generating confined ensembles.

Finally, it was possible to obtain the results for generating random variables for Weibull parameters. Overall 100 thousand values are created for sectors and their histograms are created. Generated variables are fell into 40 bins and the curve fitting their margins is a normal distribution curve. As well as, in the end of the Fortran routine means and standard deviations of observed data and generated data are compared and proved to be equal to each other.

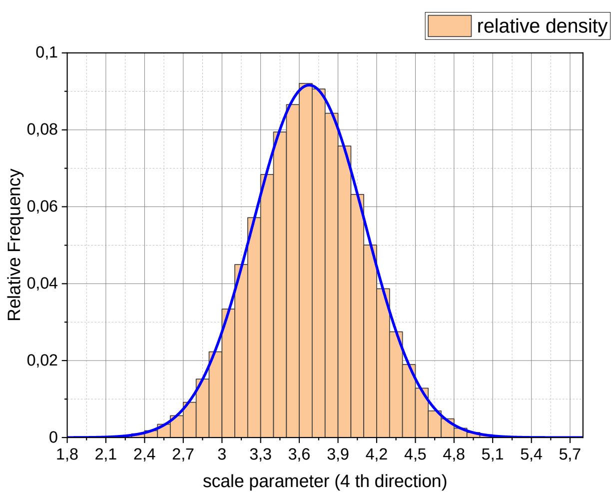


Figure 5. Histogram of scale parameter (4th direction)

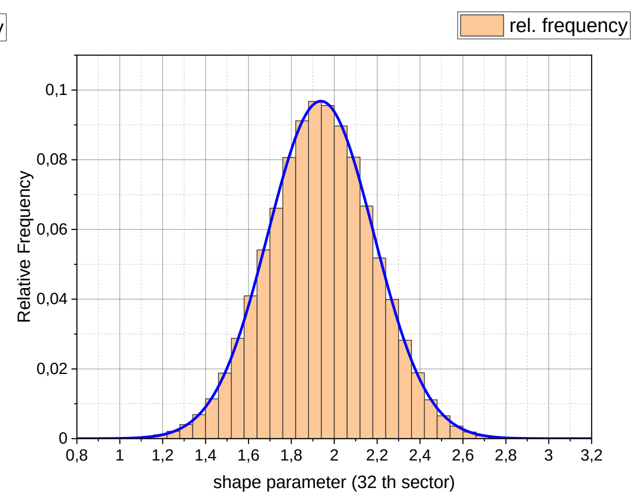


Figure 6. Histogram of shape parameter (32thdirection)

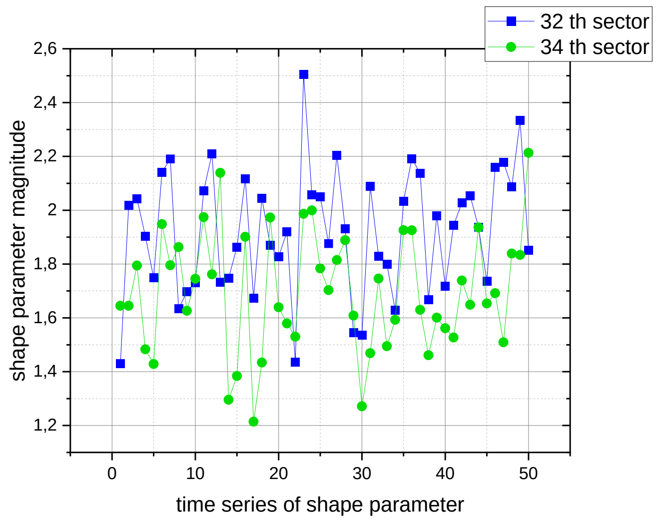


Figure 7. 50-year time series of scale parameter (32th, 34th sector)

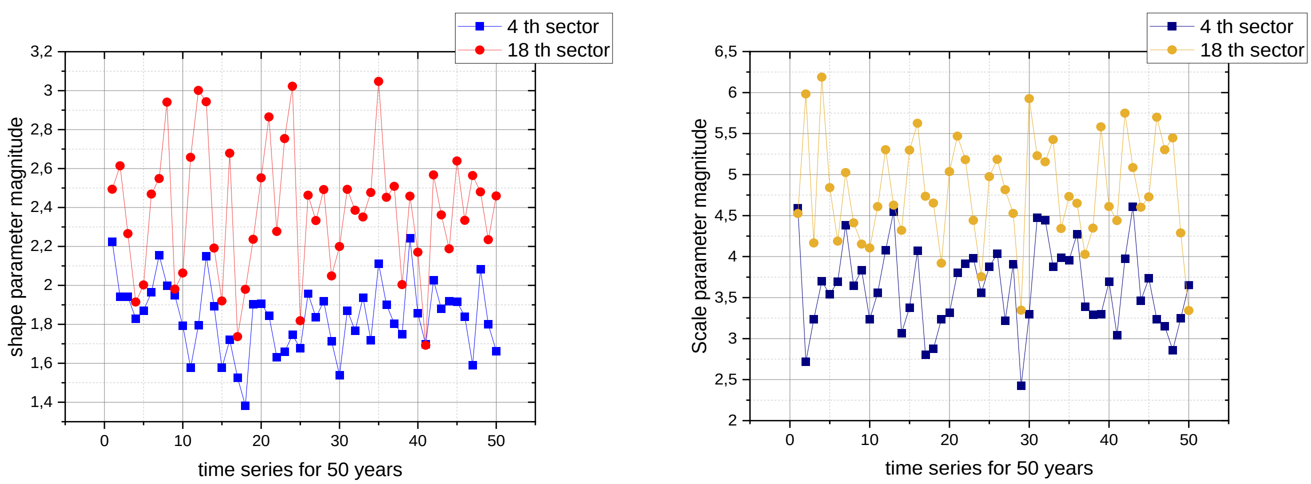


Figure 8. 50-year time series of shape and scale parameters for 4th and 18th sectors

Furthermore, the correlations are also checked and they are equal only with small differences. For example, $corr_{observed}(3,4)=0.788$ and $corr_{generated}(3,4)=0.7872$.

Creating time series is also a good way of noticing the patterns of data whether any unusual points or trends exist or not. Thus for scale and shape parameters 50-year time series plot is presented and the patterns show expected behavior range of points. There are no any significant jumps or spikes.

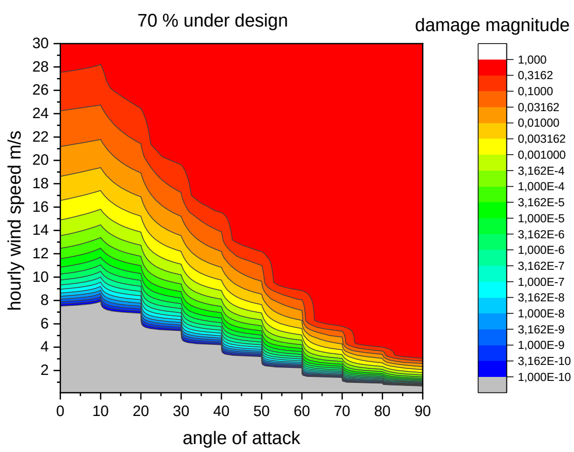
The histogram for shape parameter has been created for 32th direction sector since this is the sector not generated by simply applying Cholesky decomposition to reduced random variable matrix but the variables are generated through applying linear equations. However, its histogram also shows the similar trend and can be leveled with normal distribution curve. Similarly, time series for shape parameter is plotted for both type of sectors, reduced part and linearly dependent part. Time series of

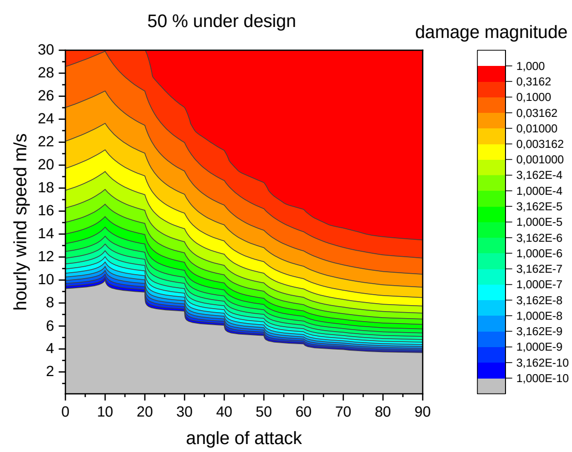
32th and 34th sectors also show no significant unusual trend and the values are within a small range radius of mean value of the both sectors.

3.5. Contour plots of damage.

To analyze the damage inflicted on the box-shaped building the wind speed level should be increased from 1 to 30 m/s. Using the load cycles and aerodynamic coefficients obtained from wind tunnel experiments it is possible to calculate the damage for every direction of wind flow. The load cycles are given from 0° to 90° as indicated and the method to find the remaining data for Tap 5 is also explained in Chapter 5.2. briefly, however, showing damage contour plots only for the first quartile is sufficient. We created some contour plots for different under design conditions to learn how the façade bolts' resistance changes under different design circumstances.

Contour plots indicate that the façade elements in the building will become very vulnerable under low design conditions making them damaged severely even at rather low wind speed levels especially when wind flow attack is from 90° degree. It is in case of 70% under design the façade elements will fail at even 4-5 m/s wind speed magnitudes when the wind flowing from 80°-90° directions.





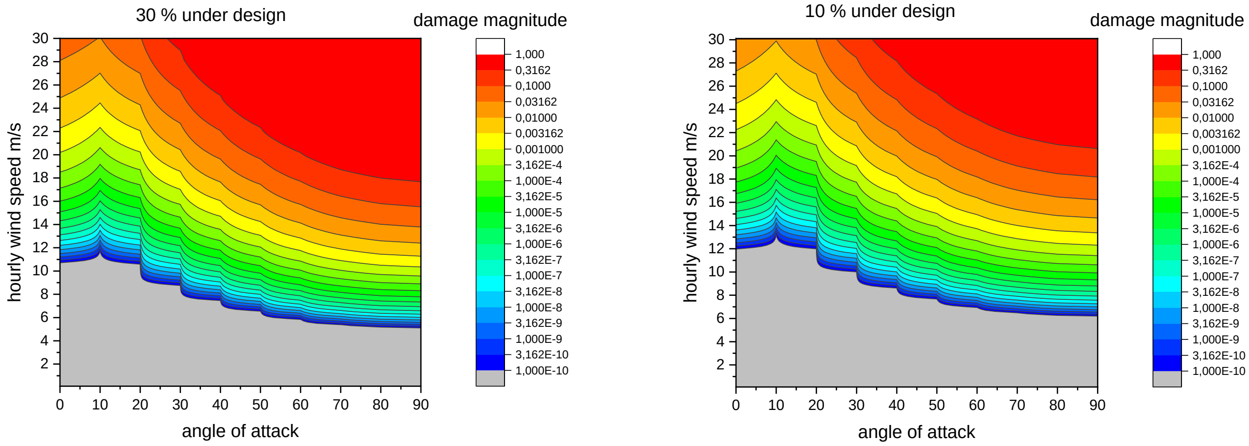
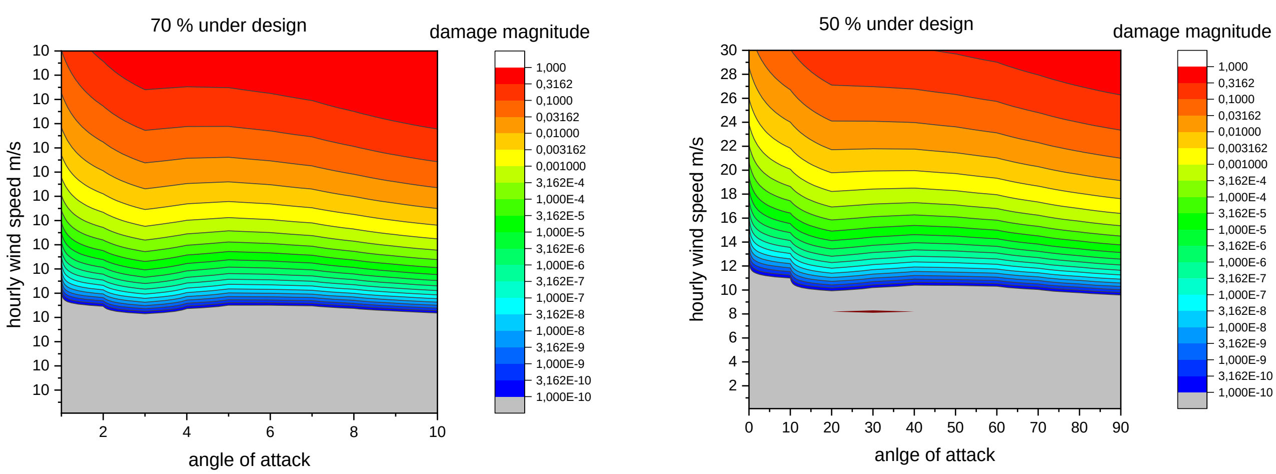


Figure 9. Damage contour plot under 70%, 50%, 30%, 10% under design conditions on Tap 5.

The next interesting tap to observe is Tap 8. In Figure 10 damage contour plots have been shown but the *cdes* coefficient was found with minimum *cpminmean, cpminsdev* values, different from equation (17)

$$cdes_{tap8} = cpminmean_{tap8} + 0.636 \cdot cpminsdev_{tap8}$$



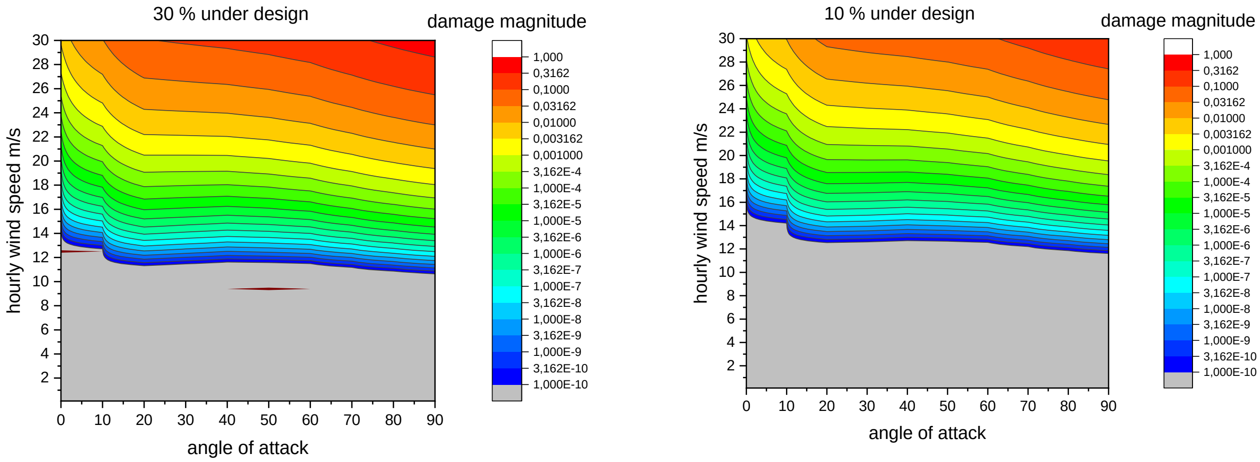


Figure 10. Damage contour plot under 70%, 50%, 30%, 10% under design conditions on Tap 8.

As it can be noticed from the graphs under such design conditions Tap 8 elements are more robust than their counterparts in Tap 5. Even with 70% under design conditions the susceptibility of façade bolts was significantly low. The directionality effects of wind flow are also not that high as in the case of Tap 5. The elements showed no damage at wind speed levels as high as roughly 11 m/s with 10 % under design case, whereas it was 6 m/s in case of Tap 5 elements. At even highest speed limit the maximum damage shown from 90^{o} direction was less than 0.3.

4. CONCLUSIONS

The main aim of the thesis was to consider the directionality effects of wind-induced fatigue damage for existing structures using the meteorological observations data in Dusseldorf from 1952 to 2017 as well as data from wind tunnel experiment implemented on box-shaped building.

The correlation matrices are crucial to learn about the relation among sectors so when generating random variables these matrices were important. 3 matrices were assembled to create 72x72 matrices to be ready for the application of Cholesky decomposition. However, here another shortcoming arises because of shortages in years to be observed. The Cholesky factorization is a scheme of decomposing matrix into lower triangular part and its conjugate transpose that perfectly works on matrices when the matrix under consideration is positive definite. However, when the size of data is less than the matrix dimension, in our case size of data for sector is 66 and matrix dimension is 72, the factorization does not work. Thus, another strategy has been used to cope with this limitation. The correlation matrix has been reduced into size where its dimension is size of observation minus one. This part of the matrix was easily decomposed as expected and random variables of 65-sized array could be generated.

In order to generate the remaining 7 variables linear equations of equations were solved. The linear weighting coefficients were found after decomposing the reduced correlation matrix into lower and upper triangles and applying forward-backward scheme. The linear weighting coefficients were used in finding the last 7 variables.

Contour plots of damage in various figures showed that in cases of under design, the façade bolts experienced more damage even at lower wind speed levels, especially the ones in the wall. Whereas, the elements on the top of box shaped building showed more resistance to damage. The directionality effects of the wind also seemed to be less significant for the top elements with little more damage when wind flowed the transverse perpendicular.

To sum up, the directionalities of wind climate to calculate fatigue damage have been observed and applicable scheme for generation of Weibull parameters have been elaborated in thesis while inconsistencies in providing such model for sector frequencies have been attempted to show. Also the damage contour plots on different tap elements have been shown.

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