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DESIGN AND IMPLEMENTATION OF A SPRING-MASS RESONATOR FOR TEACHING ASSISTANCE

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Abstract: *Over time, the issue of maintaining, restoring, and consolidating structural elements has become increasingly important for engineers and designers in the mechanical engineering sector. Many structures built in the past using outdated methods are now considered structurally unsafe according to modern standards. Replacing these structural elements is often costly and time-consuming, so strengthening has become the preferred method to enhance load capacity and extend the lifespan of these structures.*

The purpose of this work is to introduce the concept of vibration to young people and explain how exciting it is to study this field. The goal is to calculate the vibration behavior of different materials when subjected to external excitation. The method involves characterizing the resonant response of a spring-mass system that is excited by a sine-wave forcing term applied as a vertical force to the suspended mass. The prototype of the testing machine is made from second-hand materials that are readily available. The young modulus is obtained from the resonance frequency, and the non-linear coefficient is calculated using the backbone approach from resonance profile variations as a function of the forcing term amplitude. The method is highly sensitive, to the point that a maximum excitation amplitude of the order of the non-linear coefficient is required. The testing machine provides the necessary sensitivity at such small excitation amplitudes and the capability of evaluating very small damping values, as expected in high strength low damping materials. The sensitivity is ensured by an optical position sensor.

Keywords: *resonator, spring-mass system, structural elements, optical position sensor, frequency*

Introduction

This work was inspired by the unexpected convergence of two very different fields: Mechanical (structural) Engineering and Metrology of small forces, particularly for Gravitational Metrology applications. To achieve this goal, the use of innovative materials and proper reinforcement has played a crucial role in these types of applications, especially in improving the seismic behavior of masonry buildings. It is no coincidence that the first major applications in this field have been in the area of seismic retrofitting of existing structures. The use of innovative materials and proper reinforcement has allowed for the improvement of the seismic behavior of masonry buildings, making them more resistant to earthquakes and other natural disasters. This work aims to explore the potential of these materials and techniques in the field of gravitational metrology, where small forces play a critical role in the measurement of gravitational waves and other phenomena. By combining the expertise of these two fields, we hope to develop new and innovative solutions that can help us better understand the world around us.

The objective of this study is to characterize the dynamic and non-linear behavior of the most common types of materials, such as steel wire, Kevlar, nylon net, and others. The study involves designing and implementing a test machine for measuring the dynamic response of the materials, as well as an artificial damage procedure to which the materials are subjected. A critical aspect of the study is the refinement of the test equipment to eliminate any possible uncertainties in the measurements. Through these tests, weak non-linearities in the dynamic response of the different material samples were detected, and the non-linear parameters obtained were used as indicators for evaluating the long-term behavior of the materials and their sensitivity to a particular type of damage. The results of this study can be used to improve the design and performance of materials used in various applications, including structural engineering, aerospace, and defense.

While several techniques are available for finding natural frequencies and non-linearity of different materials, these techniques are often limited to materials with a small cross-sectional area, such as thin wires and yarns [1]. However, in this study, a testing machine has been developed that is applicable to materials with very thin cross-sectional areas, on the order of several microns. This testing machine allows for the investigation of the mechanical and dynamic properties of materials with larger cross-sectional areas, which is important for a wide range of applications, including structural engineering, aerospace, and defense. By using this testing machine, researchers can gain a better understanding of the behavior of materials under different conditions, which can lead to the development of new and innovative materials with improved properties and performance.

Importance of the Study of Vibration

Vibration is a crucial aspect of our daily lives, involved in activities such as hearing, seeing, breathing, and walking. Scholars have historically focused on understanding natural phenomena and developing mathematical theories to describe vibration in physical systems [2]. However, in recent times, engineers and researchers have also become interested in the practical applications of vibration, such as designing machines, structures, engines, turbines, and control systems. By studying vibration in these systems, engineers can develop more efficient and effective technologies that can improve our lives.

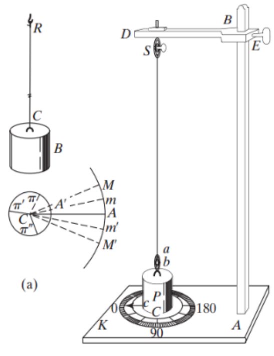

FIGURE 1. Coulomb's device for torsional vibration tests.

Overall, the study of vibration is an important and fascinating field with far-reaching implications for science, engineering, and technology.

Pythagoras conducted experiments on a vibrating string by using a simple apparatus called a monochord [3]. Pythagoras observed that if two like strings of different lengths are subject to the same tension, the shorter one emits a higher note; in addition, if the shorter string is half the length of the longer one, the shorter one will emit a note an octave above the other.

Resonance is a phenomenon that occurs when the natural frequency of vibration of a machine or structure matches the frequency of an external excitation. This can lead to excessive deflections and failure of the system. Many examples of system failures caused by resonance and excessive vibration can be found in the literature [4]. Vibration testing has become a standard procedure in the design and development of most engineering systems due to the potential devastating effects that vibrations can have on machines and structures. Earthquakes, in particular, have caused significant damage and loss of life, with over one million people having died due to earthquakes in the last centuries. The damage caused by earthquakes has resulted in several hundred billion dollars in losses in many parts of the world [5].

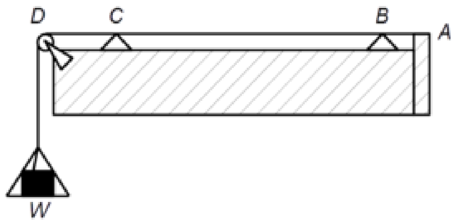

FIGURE 2. Pythagorean monochord

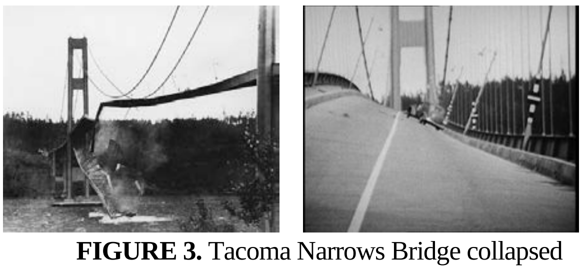

FIGURE 3. Tacoma Narrows Bridge collapsed during hard wind (The bridge opened on July 1, 1940, and collapsed on November 7, 1940).

Basic Concepts of Vibration

A vibratory system is composed of three fundamental elements: a spring or elasticity that stores potential energy, a mass or inertia that stores kinetic energy, and a damper that gradually dissipates energy [6]. The vibration of the system involves the transfer of potential energy to kinetic energy and vice versa. If the system is damped, some energy is lost in each cycle of vibration and must be replenished by an external source to maintain a state of steady vibration.

Linear and non-linear Vibration

Linear vibration occurs in a vibratory system when all of its basic components, namely the spring, the mass, and the damper, behave linearly. On the other hand, if any of these components behave non-linearly, the resulting vibration is called non-linear vibration. The differential equations that govern the behavior of linear and non-linear vibratory systems are linear and non-linear, respectively. In the case of linear vibration, the principle of superposition applies, and mathematical techniques of analysis are well established. However, for non-linear vibration, the principle of superposition is not valid, and the techniques of analysis are less well-known. As the amplitude of oscillation increases, all vibratory systems tend to behave non-linearly. Therefore, having knowledge of non-linear vibration is essential in dealing with practical vibratory systems.

Classification of Vibration

Vibration can be classified in several ways. But important classifications are followings:
free vibration is when a system vibrates on its own after an initial disturbance, without external forces. Viscous damping models fluid effects in an object. Damping causes the vibration amplitude to decrease over time, leading to equilibrium.
forced vibration is when an external force causes a system to vibrate, like in diesel engines. Resonance occurs when the frequency of the external force matches a natural frequency of the system, leading to dangerous oscillations and potential failures (see Fig. 3).

If no energy lost or dissipated on friction or other resistance during the external force applied, then the vibration is known as *undamped vibration*. If wise verse occurs, the vibration is called *damped vibration*. The major points to note from the solution are the exponential term and the cosine function. The exponential term defines how quickly the system *damps down* – the larger the damping ratio, the quicker it damps to zero. The cosine function is the oscillating portion of the solution, but the frequency of the oscillations is different from the undamped case.

$$f_d = f_n\sqrt{1-\zeta^2}$$

The frequency f_d, in this case is called the *damped natural frequency,* and it is smaller than natural frequency f_n. where ζ is *damping ratio*.

$$\zeta = \frac{c}{2\sqrt{km}}$$

More detailed in the next sections.

Deterministic and Random Vibrations

Deterministic vibration refers to the vibration of a system that is caused by a known and measurable external force or motion, which is referred to as deterministic excitation. In other words, if the magnitude and value of the excitation acting on a vibratory system is known at any given time, the resulting vibration is called deterministic vibration.

In some cases, the excitation is *non-deterministic* or *random*; the value of the excitation at a given time cannot be predicted. In these cases, a large collection of records of the excitation may exhibit some statistical regularity.

It is possible to estimate averages such as the mean and mean square values of the excitation. Examples of random excitations are wind velocity, road roughness, and ground motion during earthquakes.

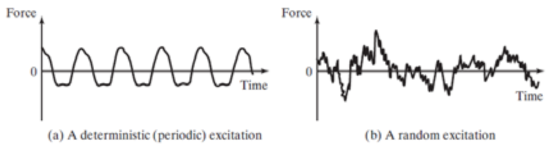

FIGURE 4. Deterministic and Random excitations

If the excitation is random, the resulting vibration is called random vibration. In this case the vibratory response of the system is also random; it can be described only in terms of statistical quantities. Figure 4 shows examples of deterministic and random excitations.

Vibration Analysis Procedure

Vibration Analysis is a technique used in industrial or maintenance settings to identify equipment faults and reduce maintenance costs and equipment downtime. It is a crucial part of Condition Monitoring programs and is often referred to as Predictive Maintenance [7]. The primary application of vibration analysis is to detect faults in rotating equipment, including fans, motors, pumps, and gearboxes. This technique can identify a range of issues, such as unbalance, misalignment, rolling element, bearing faults, and resonance conditions. A vibratory system is a dynamic system in which the variables, such as the inputs (excitations) and outputs (responses), are time-dependent. The response of a vibrating system is influenced by both the initial conditions and external excitations. Due to their complexity, it is often impossible to consider all the details of practical vibrating systems in a mathematical analysis. Therefore, only the most significant features are taken into account in the analysis to predict the behavior of the system under specific input conditions. To analyze a vibrating system, a simplified model of the complex physical system is often used to determine its overall behavior. This involves creating a mathematical model based on the physical principles that govern the system, deriving the governing equations, solving them to obtain the system's response under specific input conditions, and interpreting the results to gain insight into the system's behavior and identify any potential issues or faults [8].

The first step in analyzing a vibrating system is to create a mathematical model that represents its important features and behavior. The model should be detailed enough to describe the system in equations but not too complex. It may be linear or nonlinear, and engineering judgment is needed to create a suitable model.

Step two involves deriving equations that describe the vibration of the system using principles of dynamics. This is done by drawing free-body diagrams and isolating the mass to indicate all forces. Equations of motion are in the form of differential equations and may be linear or nonlinear. Approaches like Newton's second law, D'Alembert's principle, and conservation of energy are used to obtain the equations.

Step three involves solving the equations of motion using various techniques such as standard methods, Laplace transform, matrix methods, and numerical methods. Nonlinear and partial differential equations are challenging to solve in closed form, and numerical methods using computers can be used instead. However, drawing general conclusions about the system's behavior using computer results can be difficult.

Step four involves interpreting the results obtained from solving the governing equations. The displacements, velocities, and accelerations of the system's masses are determined. It is essential to interpret these results with a clear understanding of the analysis's purpose and the potential design implications of the results.

Spring Elements

A spring is a type of mechanical link, which in most applications is assumed to have negligible mass and damping. The most common type of spring is the helical-coil spring used in retractable pens and pencils, staplers, and suspensions of freight trucks and other vehicles. Several other types of springs can be identified in engineering applications. In fact, any elastic or deformable body or member, such as a cable, bar, beam, shaft or plate, can be considered as a spring [9]. In classical mechanics, a system that, when displaced from its equilibrium position, experiences a restoring force, *F*, proportional to the displacement, $\boldsymbol{x}$:

$$\vec{\boldsymbol{F}} = -\boldsymbol{k}\vec{\boldsymbol{x}}$$

where k is a always positive, and known as a *spring constant* or *spring stiffness* or *spring rate*. If F is the only force acting on the system, the system is called a *simple harmonic oscillator***,** and it undergoes simple harmonic motion: sinusoidal oscillations about the equilibrium point, with a constant amplitude and a constant frequency (which does not depend on the amplitude).

When the spring is stretched (or compressed) under a tensile (or compressive) force F, according to Newton s third law of motion, a restoring force or reaction of magnitude *–F (or +F)* is developed opposite to the applied force. This restoring force tries to bring the stretched (or compressed) spring back to its original unstretched or free length position.

Combination of Springs

Springs layout can be classified into two parts, one is *parallel layout*, and another is *series layout.* In figure 1.7 it can be shown. Simple example for the parallel layout (series layout) can be two springs connected in parallel (in series). In case of parallel the equivalent stiffness is obtained summing all spring stiffnesses in a sequence. In general, if we have n springs with spring constants

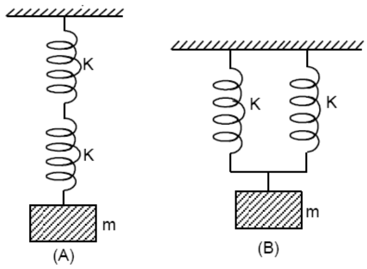

FIGURE 5. A) Springs connected in series. B) Springs connected in parallel

$k_1, k_2, \dots k_n$ in parallel, then equivalent spring constant k_{eq} can be obtained:

$$k_{eq} = k_1 + k_2 + \dots + k_n$$

As in case of springs connected in series the situation is a bit different. Again we have to consider n springs connected in series with spring constants $k_1, k_2, \dots k_n$, thus equivalent spring constant k_{eq} can be obtained:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_n}$$

Mass or Inertia Elements

The mass or inertia element is assumed to be a rigid body; it can gain or lose kinetic energy whenever the velocity of the body changes. From Newton's second law of motion, the product of the mass and its acceleration is equal to the force applied to the mass. Work is equal to the force multiplied by the displacement in the direction of the force, and the work done on a mass is stored in the form of the mass's kinetic energy.

$$F = ma = m\frac{d^2x}{dt^2} = m\ddot{x} = -kx$$

In most cases, we must use a mathematical model to represent the actual vibrating system, and there are often several possible models. The purpose of the analysis often determines which mathematical model is appropriate. Once the model is chosen, the mass or inertia elements of the system can be easily identified.

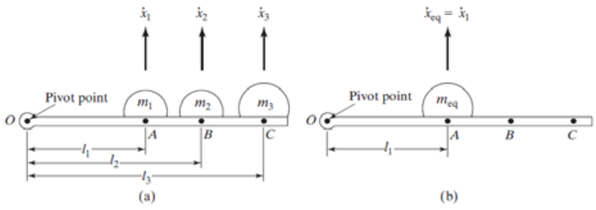

FIGURE 6. a) Distributed mass along the beam. b) Concentrated mass equivalent

In many practical applications, several masses appear in combination. For a simple analysis, we can replace these masses by a single equivalent mass. Solving differential equation above written, we find that the motion is described by the function

$$x(t) = A\cos(\omega t+\phi),$$

where,

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{\tau}.$$

Damping Elements

Damping is the mechanism by which vibrational energy is gradually converted into heat or sound due to friction or other factors. It is important to consider damping for an accurate prediction of a system's vibration response [10]. A damper is assumed to have neither mass nor elasticity, and damping force exists only if there is relative velocity between the two ends of the damper. It is difficult to determine the causes of damping in practical systems. Balance of forces for damped harmonic motions is then:

$$F=F_{ext} - kx - c\frac{dx}{dt}=m\frac{d^2x}{dt^2},$$

where c is damping coefficient. Oscillation is the repetitive variation typically in time, of some measure about a central value (often a point of equilibrium) or between two or more different states.

The term Vibration is precisely used to describe mechanical oscillation. Oscillations occur not only in mechanical systems but also in dynamic systems in virtually every area of science: for example, the beating human heart, business cycles in economics, predator-prey population cycles in ecology, geothermal geysers in geology, vibrating strings in musical instruments etc [11].

Pendulum based construction helps students to dive into vibration world. Simple model used in vibration was spring with stiffness $\underline{k}$, and the mass $\underline{m}$. Under the stiffness implied not only springs but also any object which has internal stiffness. All models can be resembled by simplified model of mass-spring-damper. Equation of motion for the model is:

$$m\ddot{x} + C\dot{x}+kx = F$$

F: resultant force acting on the object

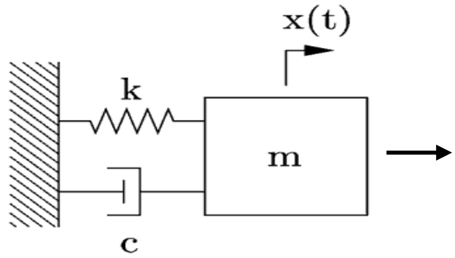

FIGURE 7. Spring – mass – damper system

Vibration testing

Vibration testing is accomplished by introducing a forcing function into a structure, usually with some type of shaker. Alternately, a DUT (device under test) is attached to the *table* of a shaker. Vibration testing is performed to examine the response of a device under test (DUT) to a defined vibration environment [12]. The measured response may be *fatigue life, resonant frequencies* or *squeak* and *rattle sound output* (NVH). Squeak and rattle testing is performed with a special type of quiet shaker that produces very low sound levels while under operation.

For relatively low frequency forcing, *servohydraulic* (electrohydraulic) shakers are used. For higher frequencies, *electrodynamic* shakers are used. Generally, one or more "input" or "control" points located on the DUT-side of a fixture is kept at a specified acceleration. Other "response" points experience maximum vibration level (resonance) or minimum vibration level (anti-resonance). It is often desirable to achieve anti-resonance to keep a system from becoming too noisy, or to reduce strain on certain parts due to vibration modes caused by specific vibration frequencies.

The most common types of vibration testing services conducted by vibration test labs are *Sinusoidal* and *Random*. Sine (one-frequency-at-a-time) tests are performed to survey the structural response of the device under test (as in our case). A random (all frequencies at once) test is generally considered to more closely replicate a real-world environment, such as road inputs to a moving automobile.

Definitions and Terminology

Cycle is a movement of a vibrating body from its undisturbed or equilibrium position to its extreme position in one direction, then to the equilibrium position, then to its extreme position in the other direction, and back to equilibrium position.

Amplitude - the maximum displacement of a vibrating body from its equilibrium position.

$$X = \frac{F_0}{k}\frac{1}{\sqrt{(1-r^2)+(2\zeta r^2)}}$$

Period of Oscillation (τ) - the time taken to complete one cycle of motion.

$$\tau = \frac{2\pi}{\omega}$$

Frequency of oscillation (f) - the number of cycles per unit time.

$$f = \frac{1}{\tau} = \frac{\omega}{2\pi}$$

Phase angle (ϕ) - the angle between two synchronous oscillations, no need to have the same amplitude.

$$\phi = \arctan\left(\frac{2\zeta r}{1-r^2}\right),$$

where r is defined as the ratio of the harmonic force frequency over the undamped natural frequency of the mass-spring-damper model.

$$r = \frac{f}{f_n}$$

Natural frequency - the frequency with which it oscillates without external forces.

$$f_n = \frac{k}{m}$$

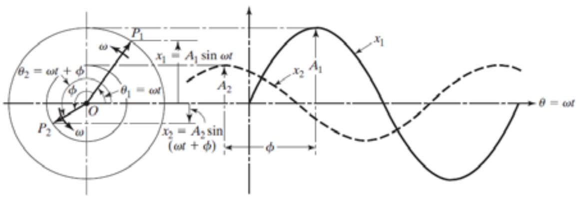

FIGURE 8. Phase difference between two vectors.

Modeling and Construction

To analyze the behavior of specimens from a dynamic point of view, it's necessary to build test equipment specifically designed and manufactured (using recycled materials identified in the laboratory, workshops, and bazaars). This chapter will describe the instrument in detail, including its various components, characteristics, and functions.

With regard to the analysis on the specimen is chosen a configuration of the testing tool which provides a beam resting on two columns, integral with a solid square stand. All difficult models which seem to be unsolvable can be divided into small simplified layouts similar to above presented. By this elementary layout visions about vibration world become easy.

As can be seen from the schematic representation shown in Figure 5.2, the upper beam has a square section dimension of 25 x 25 mm and has a maximum length of 400 mm.

The columns are formed by two square section channels, in iron, having dimensions of 30x30 mm, by 2 mm thickness and a height of 700 mm. The whole rests on a solid marble base, of dimensions 360 x 360 mm, having a thickness equal to 80 mm. As a basement we used marble with a thickness 80 mm, which guarantees good damping and very stable conditions (see fig. 10). Flat surface of the marble provides good measurement results and accurate outcomes. Firstly, we took it with sizes 650 mm by 360 mm, we found out it too big and dull, and it was difficult for movements of the model.

Because the purpose of the construction is education and model can be taken out from laboratories to classes.

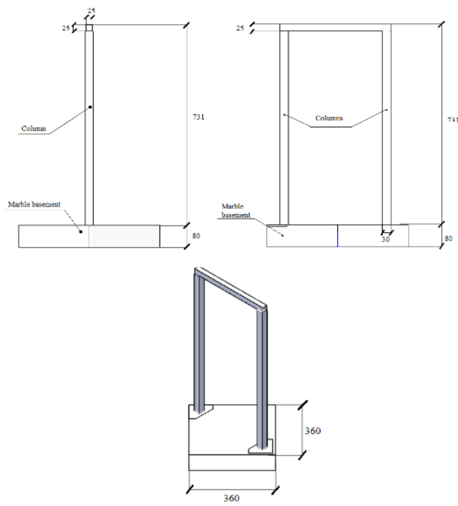

FIGURE 9. Representation of the structure – front, lateral and isometric view

The upper part of the arc must be parallel to the marble, in order simultaneously adjust both marble and arc (Fig. 15). This guarantees verticality of the hanged rope. Other distortion can affect to our measurements so every small thing must be considered. The materials used in the construction of the instrument have to be chosen so as to minimize the effect of possible temperature variations within the laboratory. So, the following table shows the thermal expansion coefficients of the materials used:

Name of Materials	Thermal expansion coefficient
Iron	$11.3 \cdot 10^{-6}$m / (m·K)
Marble	$7 \cdot 10^{-6}$m / (m·K)
Steel wire	$14.4 \cdot 10^{-6}$m / (m·K)

FIGURE 10. Marble basement

Then, decided to decreasing sizes of the marble till 360 mm and obtained a square with dimensions 360 mm by 360 mm and the thickness is 80 mm. Now, this size was comfortable to the displacements and low weight of layout gives easiness in handling by teacher and students.

Given the considerable size, it was not possible to place the equipment on an anti-vibration table, but we chose to place it on a steel structure that has always function to isolate as much as possible all the elements from the ground, so as to minimize the possibility of having small movements that could affect the validity of the measures.

FIGURE 13. Marble cut inside the frame.

As we need very flat surface, we should care about if our marble is fully horizontal with an error up to 0.1~0.8 degrees. So, we should to construct something that could keep our marble horizontally besides bandy floors. Adjustable frame was the solution of the problem.

FIGURE 14. Frame's adjustable legs

The frame with four legs that keep in a distance from the floor and can be well – regulated. As shown in figure 1.14. Four elbows with thickness of 5 mm and dimensions 40mm height and external length of 380mm, welded together, can be used as the frame outside of the marble. And four adjustable legs that consist of bolts and nuts that provides flatness.

Next step will be to think about overhanging mechanism like an arc because we are using pendulum like layout. An arc with 700mm in height and 400mm width channel glued on the marble with powerful epoxy attach.

FIGURE 15. Chuck with small tube

The ropes which are used done so that is easy to change. The chucks in each end provide changeability of the ropes. As shown in figure 15. Ends of the rope glued inside of 2 mm diameter and 30 mm length cylinder tube and those tubes are fixed at the chucks. Starting from the basic definition of force we have:

$$F = ma \approx 10 \text{ N}$$

where m represents our weight, which is 1 kg. One kilogram of brass is used as a weight (Fig. 17), while a stands for acceleration of gravity, which is equal to 9.81 m/s^2. The tension induced is then determined as

$$\sigma = \frac{F}{A} = \frac{10}{0.855 \cdot 10^{-7}} = 116.95 \text{ MPa}$$

where F, the force found above. And A is a cross section of the steel wire, which we found that it is equal to $0.855 \cdot 10^{-}7$ m^2.

FIGURE 15. An arc

The steel wire we are using is 2nd string of guitar and tension for it is equal 10 kg, which is ten times bigger than our force applied. It is possible to obtain some kind of rate of work (*work ratio*), in which the wires subjected to avoid breakage, as the ratio between the generated tension from the mass hanging from the sample (σ) and to the ultimate tensile strength of material (σ_u). Thus, work ratio is equal to 10 percent.

Circular shaped brass balance stone of which the upper part drilled in the center to fasten the chucks. From the bottom, small cylinder with coil windings co-centered and glued. The weight with coil suspends just above the small magnet. The magnet itself is situated and fixed with glue on the marble. The magnet enters 8 mm to the coil but so that they do not touch one each other. Electric current going through the coil wingdings generates magnetic field and together with magnet they attract each other thus pulls the weight down making very small movements (oscillations).

By circulating the current within the wire generates a magnetic field normal to the flow lines, it is subjected to a force that can be directed upwards or downwards, in relation to the sign of the current. The force generated can be calculated by applying second law of Laplace:

$$F = B \cdot l_b \cdot I$$

As can be seen from the formula, the force *F* is proportional to the density of magnetic flux *B*, the intensity of the current *I* and the length of the copper wires *lb*, that are within the magnetic flux. The force generated produces a deformation in the specimens, to allow the photodiode to read the optical signal, which can be translated into a displacement of the specimens, was used a lens of the dimension of 8 mm, suitably arranged near the mass. It is working as a convex lens, focuses the beam to a precise marked point. The sensor reads the movement of the lens and consequently of the specimen (steel wire), in relation to the quadrant where the LED beam is focused. The photodiode is then connected to an electronic circuit that allows the reading of the signal. Oscilloscope is used for display its graphics.

In the previous chapter the instrumentation has been described used for performing the tests dynamics and its basic operation, this, however, with the progress of work, suffered a series of changes that were necessary in the face of certain problems that have emerged during the course of the measures. In particular it has been observed that our system, subject to the free oscillations or forced ones, tended to move, and that our interest is in the main direction, or the axial one, even in that transverse to the applied force, so involving other ways to vibrate. We do care about the horizontal movements and one should prevent the mass from movements from horizontal directions. Constraint must be used to the mass.

Application of Constraints

Two auxiliary columns needed to hold constraints. Their presence, as expected prevents movement of the mass out of its axis. As a constraint we used nylon fishing net. Each column features with four pulleys. The mass kept in position touching constraints in four points. But however, it can act to the measurements. Due to small contact area with mass and constraints, the tolerances can be neglected. The nylon net with two small and equal weights at the ends, hanged to the pulleys and nylon net stretched touching the main mass.

FIGURE 17. The weight

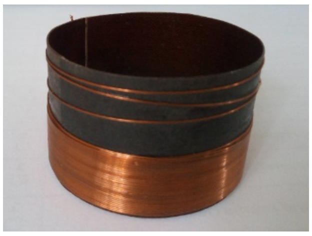

FIGURE 18. Coil

In order to detect the oscillations very sensitive sensor is used. The sensor collects light coming from the lens. The lens concentrates light which goes from LED light source and directed to the sensor which fastened on the weight.

Analyzing the System

Here we talk about the natural frequencies f_n of our system in details. What need is to calculate stiffness coefficient of our specimen under observation. In our case it is steel wire. Thus, the stiffness of the system can be obtained by following

$$k = \frac{EA_W}{l}$$

formula:

where, *E* is elastic modulus (Young's modulus) of the steel wire
A_w is cross section area of the steel wire
l is the length of our specimen

Now it is possible to obtain the frequencies that we looked for.

$$\omega_n = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad f_n = \frac{\omega_n}{2\pi}$$

TABLE 2. Properties of the system

Name	Abbrev.	Numerical value	Measuring unit
Elastic modulus of wire	*E*	$204.7 \cdot 10^9$	[N/m^2]
Wire cross section area	A_W	$0.855 \cdot 10^{-7}$	[m^2]
The length of the wire	*l*	0.48	[m]
Stiffness of the wire	*k*	36462.2	[N/m]
System mass	*m*	1	[kg]

Impulse of the system	ωn	190.9	[rad/s]
Frequency of the system	fn	30.406	[Hz]

Temperature Effect

Temperature influence is an additional aspect to take into consideration, prior analysis of external factors that may affect to our measurements. Any variations in temperature that may occur within the lab, linked to the different periods of performance of the tests and their climatic conditions, can cause a linear thermal expansion in wires. In linear expansion said that the increase in length of the body, Δl, is directly proportional to the initial length l, and the temperature variation ΔT.

$$\Delta l = \lambda \cdot l \cdot \Delta T$$

where λ is a linear thermal coefficient of material which is used. The elongation coefficient of material also can be obtained by

$$\varepsilon = \frac{\Delta l}{l}$$

This, elongation coefficient can induce inside of the wires increasing of tension $\Delta\sigma$, which leads to increase of normal stress ΔN:

$$\Rightarrow \quad \Delta\sigma = E\cdot\varepsilon \quad \Rightarrow \quad \Delta N = \Delta\sigma\cdot Aw$$

Considering possible temperature variation inside the labs ΔT, minimum as 0.5 °C and as a maximum one is 10 °C, one can obtain normal stress variation under temperature variation.

ΔT	λ	l_t	Δl_t	ε	E	$\Delta\sigma$	A_w	ΔN
[K]	[m/(m K]	[m]	[m] (*10^{-5})	-(*10^{-5})	[N/m^2]	[N/m^2] (*10^6)	[m^2]	[N]
0.5	~$15\cdot10^{-6}$	0.48	0.36	0.75	$204.7\cdot10^9$	1.54	$0.855\cdot10^{-7}$	0.131
2			1.44	3.00		6.14		0.525
3			2.16	4.50		9.21		0.788
4			2.88	6.00		12.3		1.05
5			3.60	7.50		15.4		1.31
6			4.32	9.00		18.4		1.58
7			5.04	10.5		21.5		1.84
8			5.76	12.0		24.6		2.10
9			6.48	13.5		27.6		2.36
10			7.20	15.0		30.7		2.63

TABLE 3. Normal stress change due to temperature variation

So, as temperature change effects on the wires state, thus one can calculate natural frequency change due to temperature change. Assuming that cross section of the wire remains unchanged.

ΔT	l_t	Δl_t	l_{tnew}	k	ωn	fn
[K]	[m]	[m] (*10^{-5})	[m]	[N/m]	[rad/s]	[Hz]
0.5	0.48	0.36	0,4800036	36461,91404	190,9500302	30,40605577
2		1.44	0,4800144	36461,09367	190,9478821	30,40571371
3		2.16	0,4800216	36460,54678	190,94645	30,40548567
4		2.88	0,4800288	36459,9999	190,945018	30,40525764
5		3.60	0,480036	36459,45304	190,943586	30,40502962
6		4.32	0,4800432	36458,9062	190,9421541	30,4048016
7		5.04	0,4800504	36458,35937	190,9407221	30,40457359
8		5.76	0,4800576	36457,81256	190,9392903	30,40434558
9		6.48	0,4800648	36457,26577	190,9378584	30,40411758
10		7.20	0,480072	36456,71899	190,9364266	30,40388958

TABLE 4. Influence of the temperature change to the natural frequencies

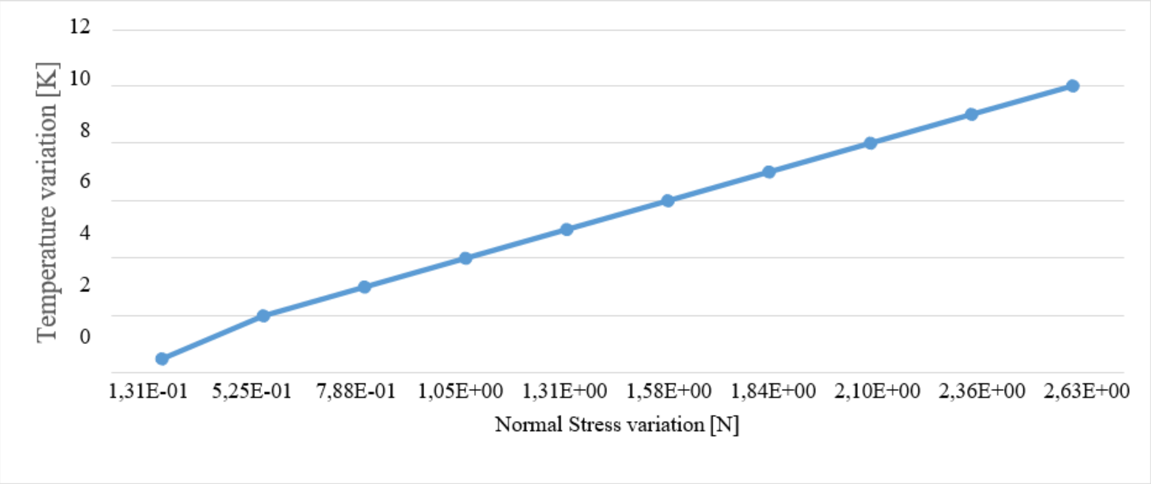

FIGURE 9. Temperature – Stress curve

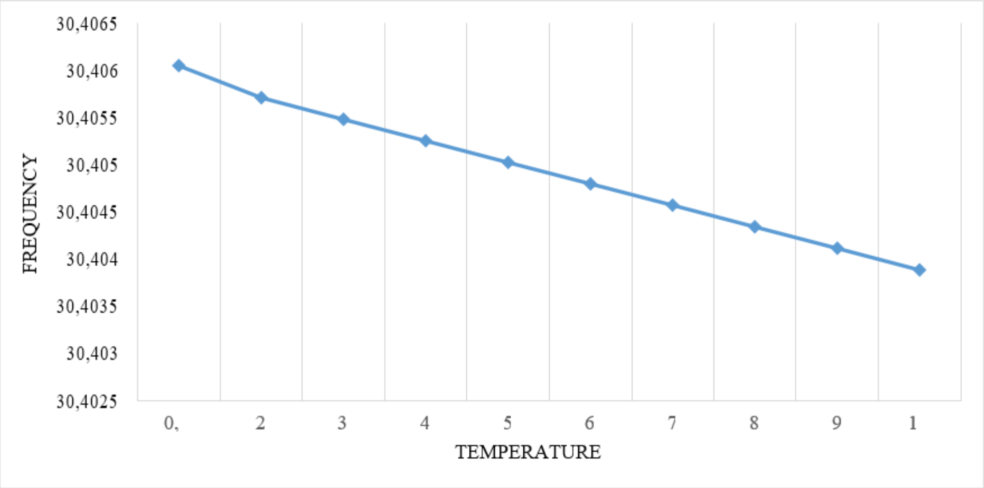

FIGURE 10. Frequency change as Temperature change.

Maximum variation of natural frequency is equal to 0.0021 Hz. This error is admissible to our measurements.

Conclusion

In conclusion, the design and implementation of a spring-mass resonator for teaching assistance is an innovative and practical approach to teaching the principles of vibration and resonance. The resonator provides a hands-on experience for students to understand the behavior of a simple mechanical system under different conditions. The design process involved selecting appropriate materials, determining the resonant frequency, and optimizing the system for maximum performance. The implementation of the resonator involved assembling the components, testing the system, and making necessary adjustments. The final product is a functional and effective teaching tool that can be used in various educational settings. Overall, the design and implementation of a spring-mass resonator for teaching assistance is a valuable contribution to the field of engineering education and provides a unique learning experience for students.

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