YEOJU TECHNICAL INSTITUTE IN TASHKENT
CENTRAL ASIAN JOURNAL OF STEM
http://stem.ytit.uz/

SECOND HARMONIC GENERATION IN BRANCHED WAVEGUIDES

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1. Introduction

Modeling of nonlinear optical processes is of practical and fundamental importance not only for optics but also relevant topics, too. One of the nonlinear processes which attracted much attention is optical harmonic generation. This plays important role for optical materials design, laser generation, signal transfer and many others. Therefore for practically important purposes it is required to develop novel functional materials and structures for optical harmonic generation, especially ways for its tuning. In simplest case, this should be second harmonic generation. In this work we propose one of such models for effective second harmonic generation in a branched optical structure, i.e., second harmonic generation in optical waveguide networks. For simplicity, we consider star branched waveguide. However, our method can be applied for arbitrary branching topologies.

2. Second-harmonic generation on the star graph

The normalized evolution equation describing optical harmonic generation in star graph branched waveguide can be written as

$$i\frac{\partial a_1^{(j)}}{\partial t}-\frac{r}{2}\frac{\partial^2 a_1^{(j)}}{\partial x^2}+\beta_j a_1^{(j)*}a_2^{(j)}e^{-i\beta t}=0,$$
$$i\frac{\partial a_2^{(j)}}{\partial t}-\frac{\alpha}{2}\frac{\partial^2 a_2^{(j)}}{\partial x^2}-i\delta\frac{\partial a_2^{(j)}}{\partial x}+\beta_j a_1^{(j)2}e^{i\beta t}=0, \qquad (1)$$

where $a_1^{(j)}$ and $a_2^{(j)}$ are the normalized amplitudes of the fundamental and the second harmonic waves, respectively, $r=-1$ for spatial solitons, $\alpha=-k_1/k_2$, and β_j are nonlinearity coefficients. Here k_1 and k_2 are the linear wave numbers, $j=1,2,3$ is the number of bonds. In all cases we set $\alpha=-0.5$.

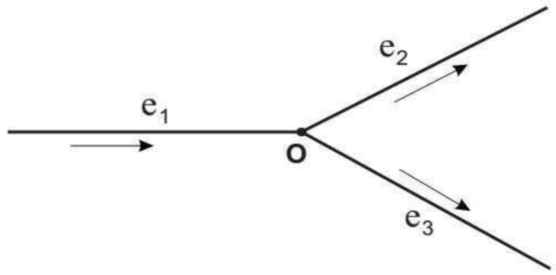


Fig. 1: Star graph

Fig.1 shows the star graph, choosing the origin of coordinates at the vertex, 0 for bond e_1 we put $x_1 \in (-\infty, 0]$ and for $e_{2,3}$ we fix $x_{2,3} \in [0, +\infty)$.

Time and coordinate variables in Eq. (1) can be separated by the following substitution:

$$a_\nu^{(j)}(x,t) = \frac{1}{\beta_j} U_\nu(\eta) \exp[i\phi_\nu(\eta, t)], \quad j = 1,2,3 \text{ and } \nu = 1,2, \qquad (2)$$

where U and ϕ are real functions, $\eta = x - vt$ is the transverse coordinate, and $\phi_\nu(\eta, t) = k_\nu t + f_\nu(\eta)$. Here v is the soliton velocity and k_ν are the nonlinear wave-numbers. To avoid all energy exchange between the waves one needs $k_2 = 2k_1 + \beta$. Then we obtain the following stationary system:

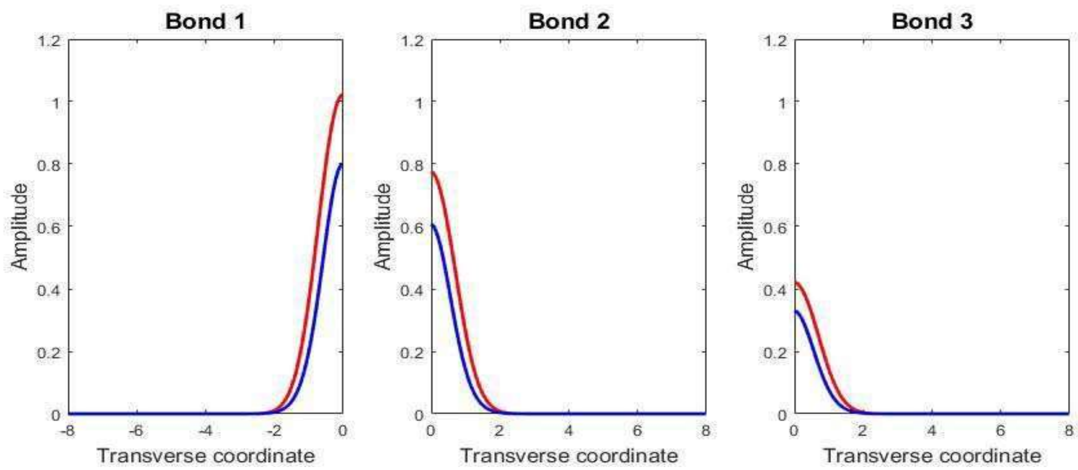
$$\frac{1}{2}\ddot{U}_1 - (k_1 - v\dot{f}_1 + \frac{1}{2}\dot{f}_1^2)U_1 + U_1 U_2 \cos(f_2 - 2f_1) = 0, \qquad (3)$$

$$\frac{1}{2}\ddot{f}_1 U_1 + (\dot{f}_1 - v)\dot{U}_1 + U_1 U_2 \sin(f_2 - 2f_1) = 0, \qquad (4)$$

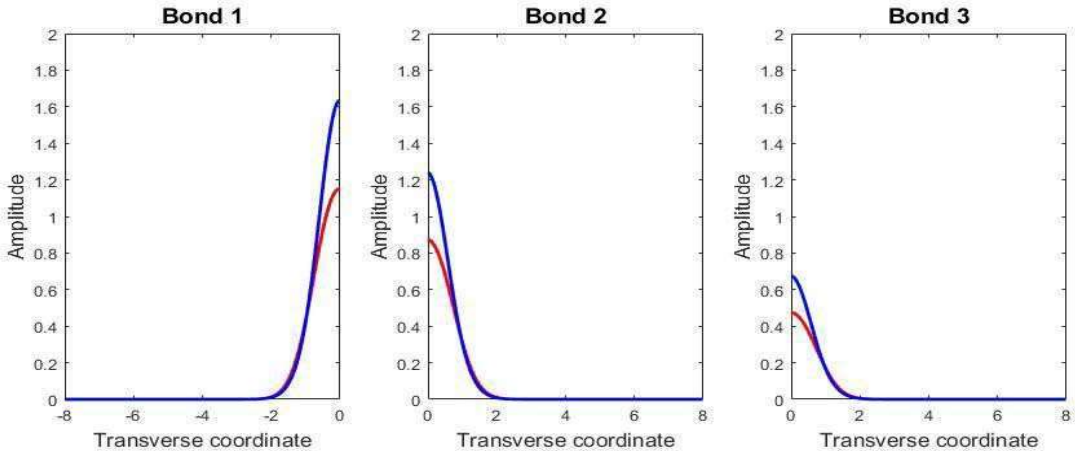
$$\frac{1}{2}\alpha\ddot{U}_2 + [2k_1 + \beta - (v + \delta)\dot{f}_2 - \frac{1}{2}\alpha\dot{f}_2^2]U_2 - U_1^2 \cos(f_2 - 2f_1) = 0, \qquad (5)$$

$$\frac{1}{2}\alpha\ddot{f}_2 U_2 + (\alpha\dot{f}_2 + v + \delta)\dot{U}_2 + U_1^2 \sin(f_2 - 2f_1) = 0, \qquad (6)$$

where the overdots indicate the derivative with respect to η.



a) $v = -0.5$



$b)\ v = 2$

Fig. 2: Amplitude profiles of the fundamental (red line) and SH (blue line) waves on the star graph. In all cases $\alpha = -0.5$, $\delta = 0$, $\beta = 0$, $k_1 = 3$ and the nonlinearity coefficients $\beta_1 = 1.66$, $\beta_2 = 2$, $\beta_3 = 3$.

To solve Eqs. (1) one needs to impose the boundary conditions at the graph vertex. They can be derived from the conservation laws for energy flow, transverse momentum and the Hamiltonian, which are given by (respectively)

$$I_j = \int_{e_j} \left(|A_1^{(j)}|^2 + |A_2^{(j)}|^2 \right) dx, \quad (7)$$

$$J_j = \frac{1}{4i} \int_{e_j} [2(A_1^{(j)*} \frac{\partial A_1^{(j)}}{\partial x} - A_1^{(j)} \frac{\partial A_1^{(j)*}}{\partial x}) + (A_2^{(j)*} \frac{\partial A_2^{(j)}}{\partial x} - A_2^{(j)} \frac{\partial A_2^{(j)*}}{\partial x})] dx, \quad (8)$$

$$H_j = -\frac{1}{2} \int_{e_j} [r | \frac{\partial A_1^{(j)}}{\partial x} |^2 + \frac{\alpha}{2} | \frac{\partial A_2^{(j)}}{\partial x} |^2 - \beta |A_2^{(j)}|^2 + \frac{i}{2} \delta (A_2^{(j)} \frac{\partial A_2^{(j)*}}{\partial x} - A_2^{(j)*} \frac{\partial A_2^{(j)}}{\partial x}) + \beta_j ((A_1^{(j)*})^2 A_2^{(j)} + (A_1^{(j)})^2 A_2^{(j)*})] dx, \quad (9)$$

where star denotes complex conjugate and $A_1^{(j)} = a_1^{(j)}$ and $A_2^{(j)} = a_2^{(j)} \exp(-i\beta t)$.

Time derivation of the conservative quantities

$$\frac{dI_j}{dt} = \mp [r \mathrm{Im}[A_1^{(j)*} \frac{\partial A_1^{(j)}}{\partial x}] + \alpha \mathrm{Im}[A_2^{(j)*} \frac{\partial A_2^{(j)}}{\partial x}] - \delta |A_2^{(j)}|^2]|_{x=0}, \quad (10)$$

$$\frac{dJ_j}{dt} = \pm [\frac{r}{2} | \frac{\partial A_1^{(j)}}{\partial x} |^2 + \frac{\alpha}{4} | \frac{\partial A_2^{(j)}}{\partial x} |^2 + \mathrm{Im}(A_1^{(j)*} \frac{\partial A_1^{(j)}}{\partial t}) + \frac{1}{2} \mathrm{Im}(A_2^{(j)*} (\frac{\partial A_2^{(j)}}{\partial t} + i\beta A_2^{(j)})) - \beta_j \mathrm{Re}(A_1^{(j)2} A_2^{(j)*})] \quad (11)$$

$$\frac{dH_j}{dt} = \mp \frac{1}{2} [2r \mathrm{Re} \left(\frac{\partial A_1^{(j)}}{\partial t} \frac{\partial A_1^{(j)*}}{\partial x} \right) + \alpha \mathrm{Re} \left(\frac{\partial A_2^{(j)}}{\partial t} \frac{\partial A_2^{(j)*}}{\partial x} \right) + \alpha\beta \mathrm{Im} \left(A_2^{(j)} \frac{\partial A_2^{(j)*}}{\partial x} \right) - \delta \mathrm{Im} \left(A_2^{(j)} \frac{\partial A_2^{(j)*}}{\partial t} \right) - \beta\delta |A_2^{(j)}|^2]|_{x=0}. \quad (12)$$

Time derivation of the total energy flow and the Hamiltonian lead to the following boundary conditions at the vertex when $\delta = 0$ and $\beta = 0$:

$$\beta_1 a_1^{(1)}|_{x=0} = \beta_2 a_1^{(2)}|_{x=0} = \beta_3 a_1^{(3)}|_{x=0},$$

$$\beta_1 a_2^{(1)}|_{x=0} = \beta_2 a_2^{(2)}|_{x=0} = \beta_3 a_2^{(3)}|_{x=0}, \quad (13)$$

$$\frac{1}{\beta_1}\frac{\partial a_1^{(1)}}{\partial x}|_{x=0} = \frac{1}{\beta_2}\frac{\partial a_1^{(2)}}{\partial x}|_{x=0} + \frac{1}{\beta_3}\frac{\partial a_1^{(3)}}{\partial x}|_{x=0},$$
$$\frac{1}{\beta_1}\frac{\partial a_2^{(1)}}{\partial x}|_{x=0} = \frac{1}{\beta_2}\frac{\partial a_2^{(2)}}{\partial x}|_{x=0} + \frac{1}{\beta_3}\frac{\partial a_2^{(3)}}{\partial x}|_{x=0}. \quad (14)$$

3. Cutoff rules

For the fundamental beam, setting $U_1 \sim \exp(-\Gamma_1\eta)$ and linearizing Eqs. (3) and (4) give:

$$\Gamma_1^2 = -\frac{2}{r}\left(k_1 + \frac{v^2}{2r}\right), \quad (15)$$
$$\dot{f}_1(\eta \to \infty) = -\frac{v}{r}. \quad (16)$$

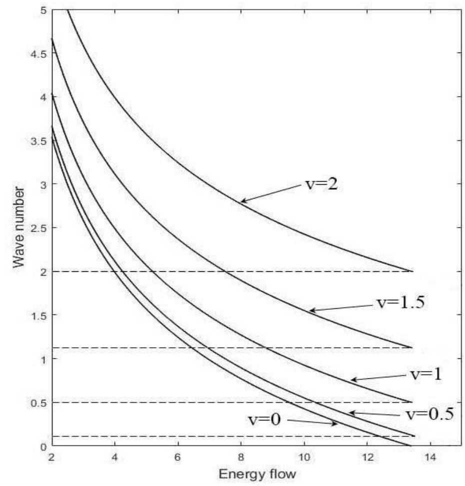


Fig. 3: Nonlinear wave-number shift versus energy flow for the families of walking solitons with different soliton velocities.

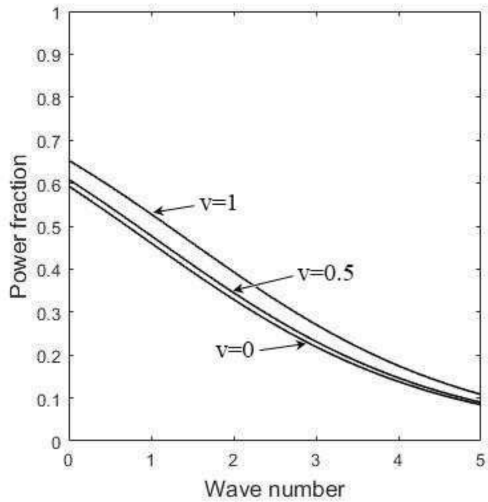


Fig. 4: Fraction of power carried by the second-harmonic beam (I_2/I) as a function of the nonlinear wave-number shift k_1 at several velocity. In all cases $\delta = 0$.

In general, the tails of the second-harmonic beam are more complicated because the nonlinear terms in Eqs. (5) and (6) may decay at $\eta \to \pm\infty$ with the same rate as the linear terms. When such is not the case, one has $U_2 \sim \exp(-\Gamma_2\eta)$, with $\Gamma_2 < 2\Gamma_1$. Then, Eqs. (5) and (6) yield

$$\Gamma_2^2 = -\frac{2}{\alpha}(2k_1 + \beta + \frac{(v+\delta)^2}{2\alpha}), \quad (17)$$

$$\dot{f}_2(\eta \to \infty) = -\frac{1}{\alpha}(v+\delta). \quad (18)$$

For the condition $\Gamma_2 < 2\Gamma_1$ to be fulfilled by Eq.(17) it is necessary that

$$\frac{1}{\alpha r}[2k_1(r-2\alpha) + \beta r + \frac{(v+\delta)^2 r}{2\alpha} - \frac{2v^2\alpha}{r}] > 0. \quad (19)$$

In the case $\alpha = -0.5$ and $r = -1$ that we always consider in the numerics here, inequality (19) leads to

$$\delta(2v+\delta) > \beta. \quad (20)$$

For given values of the various involved parameters, stationary walking solitons exist for nonlinear wave-number shifts above cutoff values:

$$k_{1,cut} = \max\{-\frac{v^2}{2r}, -\frac{1}{2}(\beta + \frac{(v+\delta)^2}{2\alpha})\}. \quad (21)$$

4. Second-harmonic generation on the H-graph

Similar as the H-graph the normalized evolution equations:

$$i\frac{\partial a_1^{(j)}}{\partial t} - \frac{r}{2}\frac{\partial^2 a_1^{(j)}}{\partial x^2} + \beta_j a_1^{(j)*} a_2^{(j)} e^{-i\beta t} = 0,$$

$$i\frac{\partial a_2^{(j)}}{\partial t} - \frac{\alpha}{2}\frac{\partial^2 a_2^{(j)}}{\partial x^2} - i\delta\frac{\partial a_2^{(j)}}{\partial x} + \beta_j a_1^{(j)2} e^{i\beta t} = 0. \quad (22)$$

H-graph is presented in Fig.4. The coordinates are defines as $x_{1,2} \in (-\infty, 0]$, $x_3 \in [0, L]$, $x_{4,5} \in [0, +\infty)$, where L is the length of bond e_3, i.e. the distance between two vertices.

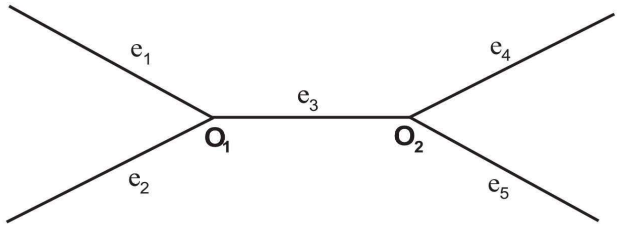


Fig. 5: H-graph

Eqs.(2)-(6) can be written for the H-graph, but here $j = 1,2,3,4,5$.

Boundary conditions at the vertices:

$$\begin{aligned}
&\gamma_1^{(1)} a_1^{(1)}|_{x=0} = \gamma_1^{(2)} a_1^{(2)}|_{x=0} = \gamma_1^{(3)} a_1^{(3)}|_{x=0},\\
&\gamma_1^{(3)} a_1^{(3)}|_{x=L} = \gamma_1^{(4)} a_1^{(4)}|_{x=0} = \gamma_1^{(5)} a_1^{(5)}|_{x=0},\\
&\gamma_2^{(1)} a_2^{(1)}|_{x=0} = \gamma_2^{(2)} a_2^{(2)}|_{x=0} = \gamma_2^{(3)} a_2^{(3)}|_{x=0},\\
&\gamma_2^{(3)} a_2^{(3)}|_{x=L} = \gamma_2^{(4)} a_2^{(4)}|_{x=0} = \gamma_2^{(5)} a_2^{(5)}|_{x=0},
\end{aligned} \quad (23)$$

$$\frac{1}{\gamma_1^{(1)}}\frac{\partial a_1^{(1)}}{\partial x}|_{x=0} + \frac{1}{\gamma_1^{(2)}}\frac{\partial a_1^{(2)}}{\partial x}|_{x=0} = \frac{1}{\gamma_1^{(3)}}\frac{\partial a_1^{(3)}}{\partial x}|_{x=0},$$

$$\frac{1}{\gamma_1^{(3)}}\frac{\partial a_1^{(3)}}{\partial x}|_{x=L} = \frac{1}{\gamma_1^{(4)}}\frac{\partial a_1^{(4)}}{\partial x}|_{x=0} + \frac{1}{\gamma_1^{(5)}}\frac{\partial a_1^{(5)}}{\partial x}|_{x=0},$$

$$\frac{1}{\gamma_2^{(1)}}\frac{\partial a_2^{(1)}}{\partial x}|_{x=0} + \frac{1}{\gamma_2^{(2)}}\frac{\partial a_2^{(2)}}{\partial x}|_{x=0} = \frac{1}{\gamma_2^{(3)}}\frac{\partial a_2^{(3)}}{\partial x}|_{x=0}, \quad (24)$$

$$\frac{1}{\gamma_2^{(3)}}\frac{\partial a_2^{(3)}}{\partial x}|_{x=L} = \frac{1}{\gamma_2^{(4)}}\frac{\partial a_2^{(4)}}{\partial x}|_{x=0} + \frac{1}{\gamma_2^{(5)}}\frac{\partial a_2^{(5)}}{\partial x}|_{x=0},$$

where $\gamma_\nu^{(j)} = \beta_j$. Fig.6 shows that the amplitudes of the fundamental and second harmonic waves on the H-graph.

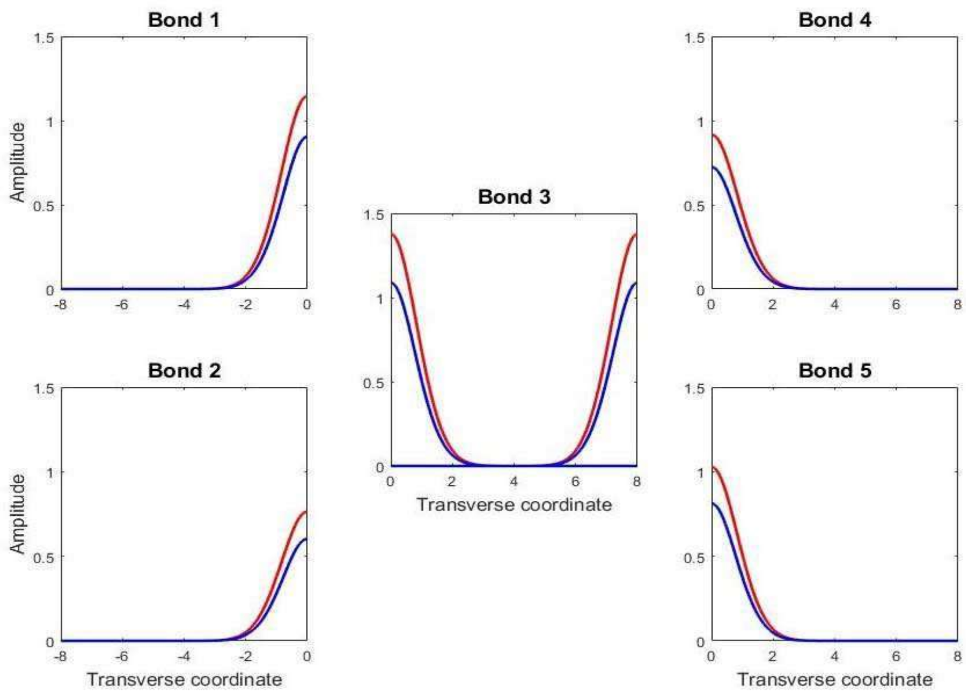


Fig. 6: Amplitude profiles of the fundamental (red line) and SH (blue line) waves on the H-graph. Where $\alpha = -0.5$, $\nu = 0.5$, $\delta = 0$, $\beta = 0$, $k_1 = 3$, $L = 8$ and the nonlinearity coefficients $\beta_1 = 2$, $\beta_2 = 3$, $\beta_3 = 1.6641$, $\beta_4 = 2.5$, $\beta_5 = 2.2299$

5. Conclusion

Solving Eqs. (1) numerically we obtain profiles of the generated second harmonics. Fig. 2 presents profile of the amplitude of generated second harmonics for different values of the parameters. Thus in this work we obtained solutions of the nonlinear system of equations describing optical second harmonic generation in branched optical waveguides. The results obtained can be used for modeling and design branched optical materials allowing tunable second harmonic generation to be used for different practical purposes.

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