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Dynamics of one-dimensional large and small static quantum droplets

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The structure and dynamics of one-dimensional binary Bose gases forming quantum droplets is studied by solving the corresponding amended Gross-Pitaevskii equation. Two physically different regimes are identified, corresponding to small droplets of an approximately Gaussian shape and large "puddles" with a prominent flat-top plateau. Small droplets collide quasi-elastically, featuring the soliton-like behavior.

Keywords: quantum droplets, quantum, soliton, spin

I. Introduction

Among the important advances in the study of cold bosonic gases and superfluids is the implementation of quantum drops in a series of experiments with anisotropic interactions between dipolar atoms [1, 2], as well as contact isotropic interactions. in two-component Bose gases. [3,4]. Therefore, the attractive and repulsive forces whose interaction leads to the formation of quantum droplets are anisotropic for dipoles and isotropic for mixtures of two-component gases. In one-dimensional geometry the (slightly) repulsive meanfield (MF) contribution to the energy per particle scales linearly with density n of the gas, getting balanced by the attractive beyond mean-field (BMF) as $-n^{1/2}$. As a result, the system's energy features a minimum, corresponding to the formation of a liquid droplet [5, 6]. Notably, its density can be tuned in a wide range, making it possible to create extremely dilute liquids and thus realize, perhaps, the most dilute liquid ever observed in any physical setting. An additional interest in this new class of quantum liquids, as compared to liquid helium, is that the condensate fraction is very large, permitting one to make accurate quantitative predictions based on the mean-field theory amended by the BMF correction. Indeed, the description in terms of the effective GrossPitaevskii equation (GPE), used to model the dipolar condensates [7], agrees with *ab initio* quantum Monte Carlo calculations for dipolar droplets [8, 9], and for ones formed in the binary BEC dominated by the contact

interactions [10]. It was argued that the quantum droplets may find an application to the design of a precise matter-wave interferometer [11, 12].

While the present study concentrates on the properties in one-dimensional geometry, we find it instructive to make a comparison with three-dimensional counterpart in terms of the sign of the BMF corrections and the value of the gas parameter where MF theory can be applied. Indeed, one-dimensional systems might seem counterintuitive for certain properties. Suppose we consider a single-component gas with delta-interacting potential $V(r) = g\delta(r)$, its potential energy per particle is $E/N = gng_2(0)/2$ where $g_2(0)$ is the value of the densitydensity correlation function $g_2(r) = (n(r)n(0))/(n)^2$ at contact position $r = 0$. The potential energy per particle scales linearly with the density. The potential energy can be reduced to zero by making the particle fully impenetrable $g_2(0) = 0$. On the other hand in one dimension the impenetrable condition induces kinetic energy per particle which scales quadratically with the density, $E/N \propto \hbar^2 n^2/m$. It means that in one dimension the mean field regime, where one can neglect correlations and set $g_2(0) = 1$, is reached for large density. Here the meanfield energy n becomes smaller than n^2 dependence of a strongly correlated (Tonks-Girardeau) gas which is obtained when $g_2(0) = 0$ (Pauli exclusion). This is exactly on the opposite from the "usual" three dimensional situation where the mean-field energy $\propto n$ becomes energetically preferable at small densities compared to the kinetic energy per particle due to Pauli principle $\propto \hbar^2 n^{2/3}/m$. As a result, the regimes of the applicability of the meanfield theory are swapped and correspond to small (3D) and large (1D) densities.

This possibility provides an important interdisciplinary connection to the field of nonlinear optics [15], as concerns the underlying model equations with higher-order nonlinearities [16] and, possibility, controlled generation of solitons in these systems. On the other hand, in the case when two-component features in the dynamics are essential, they may be affected by an additional linear interconversion between the components [17].

In three and two dimensions, quasi-1D solitons are unstable with respect to the transverse snake instability, although the stability can be enhanced by imposing rotation to the quantum droplets [18]. The advantage of the proper 1D geometry, imposed by the tight confinement in the transverse directions (cf. the experimental realization of the Tonks-Girardeau gas [19, 20]), is that such an instability is absent, thus permitting one to realize a very clean and highly controllable many-body testbed which may permit the measurement of quantum many-body effects with very high precision.

Commonly known hallmarks of solitons are being (i) self-trapped and (ii) robust with respect to soliton-soliton collisions. While the former feature is definitely present in quantum droplets, the latter one should be yet verified. It was proposed to use Gaussian *ans¨atze* for gaining an analytical insight in physics of dipolar [7] and BEC [3] droplets. In particular, the dynamical version of the Gaussian-based variational approximation (VA) can be used to predict the frequency of intrinsic oscillations of the soliton-like objects in an excited state [21–22].

II. Model systems limit cases of small snd large drops

We consider the binary BEC with mutually symmetric spinor components, assuming that the coupling constants describing the repulsion between the atoms in each one are equal, $g_{\uparrow\uparrow} = g_{\downarrow\downarrow} \equiv g$, and numbers of atoms in the components are equal too. In this case, the equilibrium densities of both components are identical, which makes the analysis essentially easier, and results clearer.

The underlying time-dependent GPE for the onedimensional droplet with the symmetric components is [5]

$$i\hbar\psi_t = -\frac{\hbar^2}{2m}\psi_{xx} + \delta g|\psi|^2\psi - \frac{\sqrt{2m}}{\pi\hbar}g^{\frac{3}{2}}|\psi|\psi\,, \qquad (1)$$

where parameters δg and g are positive and are related to the coupling constants in the two spinor components as $\delta g = (g_{\uparrow\downarrow} + g_{\uparrow\uparrow}g_{\downarrow\downarrow})^{1/2} > 0$ and $g = (g_{\uparrow\uparrow}g_{\downarrow\downarrow})^{1/2}$. The coupling constant g is relevant for inducing a hard "spin" mode while the difference δg between attractive intercomponent and repulsive intracomponent interactions is responsible for appearance of a soft "density" mode and condition $\delta g \ll g$ induces a separation of scales.

In experiments it is possible to tune δg both to positive or negative values. The proper sign is chosen in such a way that the imbalance in the mean-field terms is opposite to the beyond mean-field contribution and consequently depends on dimensionality of the problem. In one dimension, the beyond mean-field terms are directly obtained from the second-order perturbation theory which produces a negative correction to the energy [14]. Accordingly, a positive mean-field imbalance is needed, $\delta g > 0$, for producing am energy minimum in the equation of state. In 3D, the BMF term includes the renormalization correction [13] to the scattering amplitude within the second Born approximation, resulting in the positive LHY term and requiring $\delta g < 0$ [5].

We define characteristic units of length x_0, time t_0 and energy E_0:

$$x_0 = \frac{\pi\hbar^2\sqrt{\delta g}}{\sqrt{2}mg^{3/2}}\,, (2)$$

$$t_0 = \frac{\pi^2\hbar^3\delta g}{2mg^3}, \quad (3)$$

$$E_0 = \frac{\hbar^2}{mx_0^2} = \frac{\hbar}{t_0} = \frac{2mg^3}{\pi^2\hbar^2\delta g}, \quad (4)$$

which yield a characteristic factor for the normalization of the wave function,

$$\psi_0 = \frac{\sqrt{2m}}{\pi\hbar\delta g}\, g^{3/2}. \quad (5)$$

We demonstrate below that

$$N_0 = \psi_0^2 x_0 = \frac{\sqrt{2}}{\pi}\left(\frac{g}{\delta g}\right)^{3/2}. \quad (6)$$

determines a critical number of particles separating two different physical regimes.

Thus, rescaling

$$t = t_0 t', \; x = x_0 x', \; \psi = \psi_0 \psi', \quad (7)$$

casts Eq. (1) in an equation without free coefficients (where the primes are omitted):

$$i\psi_t + \frac{1}{2}\psi_{xx} - |\psi|^2\psi + |\psi|\psi = 0, \quad (8)$$

A peculiarity of the 1D geometry is that the groundstate solution of the GPE for the droplet, Eq. (8), can be found in an explicit form [9]:

$$\psi_{exact}(x) = -\frac{3\mu\exp(-i\mu t)}{1+\sqrt{1+\frac{9\mu}{2}}\cosh\left(\sqrt{-2\mu x^2}\right)}, \quad (9)$$

with the relation between normalization N and chemical potential μ given by

$$N = \frac{4}{3}\left[\ln\left(\frac{\sqrt{-\frac{9}{2}\mu+1}}{\sqrt{\frac{9}{2}\mu+1}}\right) - \sqrt{-\frac{9}{2}\mu}\right]. \quad (10)$$

The equilibrium density corresponding to the spatially uniform state ($N \to \infty$), and the respective chemical potential, in units defined by Eqs. (2) and (4), are

$$\frac{n_0}{\psi_0^2} = \frac{4}{9}, \quad (11)$$

$$\frac{\mu_0}{E_0} = -\frac{2}{9}, \quad (12)$$

In a large finite-size droplet ("puddle"), μ approaches the constant value (12) corresponding to the chemical potential of an infinitely extended uniform liquid at zero pressure. The chemical potential (10) is expanded as

$$\mu = -\frac{2}{9} + \frac{8}{9}\exp\left(-2 - \frac{3}{2}N\right), \quad (13)$$

and features an exponentially weak dependence on N. On the other hand, for small droplets with small N the dependence has a power-law form:

$$\mu = -\frac{1}{2^{\frac{1}{3}}3^{\frac{2}{3}}}N^{\frac{2}{3}} = -0.382N^{\frac{2}{3}}, \quad (14)$$

In this case, the dependence on N is much stronger, as long as $|\mu|$ is small.

The total energy E can be obtained by integrating the chemical potential, $E(N) = \int_0^N \mu(N')dN'$. For small N, Eq. (14) results in a power-law dependence,

$$E = -\frac{1}{5}\left(\frac{3}{2}\right)^{\frac{1}{3}} N^{\frac{5}{3}} = -0.229N^{\frac{5}{3}}, \qquad (15)$$

while Eq. (13) produces an asymptotically linear dependence on large N:

$$E = -\frac{6}{27}N + \frac{16\exp(-2)}{27} - \frac{16}{27}\exp\left(-2 - \frac{3}{2}N\right). \qquad (16)$$

A typical size of the droplet can be easily estimated in both limits. The large droplet includes a bulk (*flat-top*) region with the nearly uniform density given by Eq. (11), with size $L = N/n_0$. The respective mean-square size also increases linearly with the number of particles,

$$\sqrt{\langle x^2 \rangle} = \frac{L}{2\sqrt{3}} = \frac{N}{2\sqrt{3}n_0} = 0.65\,. \qquad (17)$$

III. Conclusion

The main results reported in this paper are summarized as follows. We have studied static and dynamical properties of 1D two-component Bose gases forming quantum droplets, in the framework of the mean-field theory (GPEs) amended by the BMF (beyond-meanfield) corrections. Lines correspond to different values of the Weber number, defined as per $N \ll 1$ 1 have an approximately Gaussian shape, being well described by the corresponding VA (variational approximation). Collisions between small droplets do not essentially alter their shape, hence droplets may be considered as solitons in a nearly integrable setting.

On the other hand, large "puddle" droplets with $N \gg 1$ 1 feature a top-flat density profile, with an approximately constant density corresponding to its equilibrium value in the uniform liquid. Although the VA fails to describe the exponential decay of the density profile at large distances, it is quite precise for small droplets and even produces meaningful results for a number of quantities of the "puddle" droplets. We have observed splitting and merger in collision of such extended droplets, depending on the collision velocity. We have produced the stability diagram for a single droplet with respect to imprinting a spatially periodic density modulation onto it. It demonstrates a fragmentation threshold in large (broad) droplets, with the critical Weber number ~ 1.

As an extension of the present work, it may be interesting to verify the validity of the mean-field theory, amended by the BMF terms, for predicting energies, density profiles and frequencies of oscillations, by means of the quantum Monte Carlo technique. In particular, it will be relevant to check if the entrainment between two

superlfuid components, known as Andreev-Bashkin effect [39, 40], can be observed in intrinsic oscillations of the droplets.

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